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Chemical Engineering
15/ENG01/001
ENG 381

2. $y = x^3 e^{4x}$

find y^5

Solution

$u = e^{4x} \qquad v = x^3$

$$y^5 = u^5 v + 5u^4 v' + 10u^{(5)} v^2 + 10u^{(2)} v^{(3)} + 5u' v^4 + u v^5$$

$$y^n = u^{(n)} v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \cdots$$

differentiating u and v

$u = e^{4x} \qquad v = x^3$

$x' = 4e^{4x} \qquad v' = 3x^2$

$u'' = 16e^{4x} \qquad v'' = 6x$

$u^3 = 64e^{4x} \qquad v^{3} = 6$

$u^4 = 256e^{4x} \qquad v^{4} = 0$

$u^5 = 1024e^{4x} \qquad v^5 = 0$

$$y^5 = 1024e^{4x} \cdot x^3 + \cancel{5(e^{4x})^{(5-1)} 3x^2}\, 5(256e^{4x})\, 3x^2 + \frac{5(5-1)}{2!} \cdot 64e^{4x} \cdot 6x + \frac{5(5-1)(5-2)}{3!} \cdot 16e^{4x}\, 6 + \frac{5(5-1)(5-2)(5-3)}{4!} 4e^{4x}(0)$$

$$y^5 = 1024e^{4x} x^3 + 3840e^{4x} x^2 + \frac{5(4)}{2!} 64e^{4x} + \frac{5(4)(3)}{3!} 96e^{4x} + \frac{5(4)(3)(2)}{4!} 4e^{4x}(0)$$

$$y^5 = 1024e^{4x} x^3 + 3840e^{4x} x^2 + 3840e^{4x} + 960e^{4x} + 0.$$

3. $x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$

Solution

$$\overbrace{x^2 y''}^{w_1} + \overbrace{x y'}^{w_2} + \overbrace{y}^{w_3} = 0$$

$u = y^2$ $\qquad\qquad$ $u = y'$

$u^n = y^{(n+2)}$ $\qquad\qquad$ $u^n = y^{(n+1)}$

$u^{(n-1)} = y^{(n+1)}$ $\qquad\qquad$ $u^{(n-1)} = y^{(n)}$

$u^{(n-2)} = y^{(n-1)}$

$v = x^2$ $\qquad\qquad$ $v = x$

$v' = 2x$ $\qquad\qquad$ $v'' = 0$

$v'' = 2$

$v''' = 0$

$$w_1^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2!} y^{(n)} \cdot 2 + 0$$

$$w_2^{(n)} = y^{(n+1)} x + n y^{(n)} \cdot 1 + 0$$

$$w_3^{(n)} = y^{(n)}$$

$$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{n+1} + ny^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + y^{n+1}[2xn + x] + y^n[n(n-1) + n + 1] = 0$$

$$x^2 y^{(n+2)} + y^{(n+1)}[x(2n+1)] + y^n[n^2 - n + n + 1] = 0$$

$$x^2 y^{n+2} + (2n+1) x y^{n+1} + y^n[n^2+1] = 0$$

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1. $y = e^{x^2} + x$

$y' = (2x+1) e^{x^2+x}$

$y'' = (2x+1)(2x+1)(e^{x^2+x}) + 2e^{x^2+x})$

$y'' = (2x+1)(2x+1) \quad (e^{x^2+x}) + 2(e^{x^2+x})$

$y = e^{x^2+x} \quad y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(y^{(1)}) + 2(y^{(0)})$

2. $y'' = 2x+1(y^{(1)}) + 2(y^{(0)})$

$y'' - 2x+1(y^{(1)}) - 2(y^{(0)}) = 0$

$\underbrace{y^{(2)}}_{w_1} - \underbrace{2x+1}_{w_2}(y^{(1)}) - 2(\underbrace{y^{(0)}}_{w_3}) = 0$

$w_1^{(n)} = y^{(n+2)}$

$w_2^{(n)} = y^{(n+1)} \, 2x+1) + ny^{n}(-2)$

$w_3 = y^{(n)}$

$y^{(n+2)} - [y^{(n+1)}(2x+1) + ny^{n}(2)] - 2y^{(n)}$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) - 2ny^{n} - 2y^{(n)}$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) - [2ny^{n} + 2y^{(n)}]$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) - 2y^{(n)}[n+1]$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}[n+1]$

$y^{(n+2)} = 2x+1(y^{(n+1)}) + 2(n+1)[y^{(n)}]$