[illegible]

1) $y = e^{x^2+x}$

soln

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

hence

$y'' = y'(2x+1) + 2y$

$y^{(n+2)} = y^{(n+1)}\cdot(2x+1) + (n+1)y^n \cdot 2$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n.$

2) $y = x^3e^{4x}$

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}v^3 + \dots +$$

Let $x^3 = v$ and $u = e^{4x}$

$y^5 = (4)^5 e^{4x}\cdot x^3 + 5(4)^4e^{4x}\cdot 3x^2 + 10(4)^3e^{4x}\cdot 6x + 10(4)^2e^{4x}\cdot 6 + 0$

$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x} + 0$

$y^5 = e^{4x}[1024x^3 + 3840x^2 + 3840x + 960].$

3) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$x^2y^{(2)} + xy' + y = 0$

using leibnitz theorem

$$y^n = y^{(n+2)}\cdot x^2 + n\cdot 2xy^{(n+1)} + \frac{2n(n-1)y^n}{2!} + y^{(n+1)}\cdot x + ny^n + y^n$$

$y^n = x^2y^{(n+2)} + 2xy^{(n+1)}n + n(n-1)y^n + xy^{(n+1)} + ny^{(n)} + y^{(n)}$

$y^n = y^{(n+2)}(x^2) + y^{(n+1)}(2xn + x) + y^{(n)}(n(n-1) + n + 1)$

$y^n = x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2-1)y^n.$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-1)y^n = 0.$