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ENG381

1 If $y = e^{x^2+x}$ show that $y^n = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2 Using the Leibnitz theorem given that

i $y = x^3 e^{4x}$ determine $y^{(5)}$

ii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

SOLUTION

1 If $y = e^{x^2+x}$

$u = e^{x^2+x}$ $\quad v = 1$

$u^n = (2x+1)^n e^{x^2+x}$ $\quad v^{(1)} = 0$

$u^{(n-1)} (2x+1)^{n-1} e^{x^2+x}$

$y^n = u^n v + n u^{(n-1)} v^{(1)}$

$y^n = (2x+1)^n e^{x^2+x} \cdot 1 + (2x+1)^{n-1} e^{x^2+x} \cdot 0$

$y^n = (2x+1) e^{x^2+x}$

let $n = 1$

$y^{(1)} = (2x+1)^1 e^{x^2+x}$; $u = e^{x^2+x}$; $v = 2x+1$

$u^{(n)} = (2x+1)^n e^{x^2+x}$ $\quad v^{(1)} = 2$

$u^{(n-1)} (2x+1)^{n-1} e^{x^2+x}$ $\quad v^{(2)} = 0$

$= n = 1$

$y'' = (2x+1)(2x+1) e^{x^2+x} + 2(2x+1)^0 \cdot e^{x^2+x}$

$y'' = (2x+1)(2x+1)' e^{x^2+2} + 2(e^{x^2+x})$

Substitute $y = e^{x^2+x}$ and $y' = (2x+1)' e^{x^2+x}$ in y''

$y'' = (2x+1)y' + 2y$

$y'' = y'(2x+1) + 2y$

ii $y^{(2)} = y^{(1)}(2x+1) + 2y$

$y^2 = y^{(1)}(2x+1) - 2y = 0$

$W_1 = y^{(2)}$

$W_2 = y^{(1)}(2x+1)$

$W_3 = -2y$

$u = y^{(2)}$ $\quad v = 1$

$u^n = y^{(n+2)}$ $\quad v^{(1)} = 0$

$u = -y^{(1)}$ $\quad v = 2x+1$

$u^n = -y^{(n+1)}$ $\quad v^{(1)} = 2$

$u^n = -y^n$ $\quad v^{11}$ 0

$u = y$ $\quad v = 2$

$u^n = -y^n$ $\quad v' = 0$

$y^{(n)} = 0 \quad u^{(n)} v + n u^{(n-1)} v^{(1)}$

$y^{(n)} = 0 \quad y^{(n+2)}(1) - y^{(n+1)}(2x+1) + n(-y^n)(2 + 2(-y^n))$

$y^{(n)} = 0 \quad y^{(n+2)} - (2x+1) y^{(n+1)} - 2xn(y^n) - 2(y^n)$

$y^{(n)} = 0 \quad 2n(y^n) + 2(y^n) + (2x+1) y^{(n+1)} = 0 \quad y^{(n+2)}$

$y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^{(n)}$

Using the lebnitz theorem.

$y = x^3 e^{4x}$ determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v^{(1)} = 3x^2$

$u^{(n-1)} = 4(n-1) e^{4x}$; $\quad v^{(2)} = 3x^2$

$u^{(n-2)} = 4(n-2) e^{4x}$; $\quad v(3) = 6x$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$; $\quad v^{(4)} = 0$

$$y^{(n)} = u^{n} v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2) u^{(n-3)}}{3!} v^3$$

$$y^{(n)} = 4^n e^{4x} \cdot x^3 + n \cdot 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2 \times 1} 4^{(n-2)} e^{4x} \cdot 6x + \cdots$$

$$+ \cdots \frac{n(n-1)(n-2)}{3 \times 2 \times 1} 4^{(n-3)} e^{4x} \cdot 6$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{(4)} e^{4x} \cdot 3x^2 + \frac{5(4)}{2 \times 1} 4^{(3)} e^{4x} \cdot 6x + \cdots$$

$$+ \cdots \frac{5(4)(3)}{3 \times 2 \times 1} \times 4^{(2)} e^{4x} 6$$

$y^{(5)} = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$

$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

$y^{(5)} = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$

2b $x^2 y'' + xy' + y = 0$; $x^2 y^{(2)} + x y^{(1)} + y^{(0)} = 0$

$W_1 = x^2 y^{(2)}$ $\quad V_2 = x y^{(1)}$ $\quad W_3 = y^{(0)}$

$U = y^{(2)}$ $\quad V = x^2$

$U^n = y^{(n+2)}$ $\quad V' = 2x$

$U^{(n-1)} = y^{(n+1)}$ $\quad V'' = 2$

$U^{(n-2)} = y^{(n)}$ $\quad V''' = 0$

$U = y^{(1)}$ $\quad V = x$

$U^n = y^{(n+1)}$ $\quad V' = 1$

$U^{n-1} = y^{(n)}$ $\quad V'' = 0$

$U = y^{(0)}$ $\quad V = 1$

$U^n = y^n$ $\quad V^{(1)} = 0$

$y^n = y^{(n+2)} \cdot x^2 + n \cdot y^{(n+1)} \cdot \underline{2x + n(n-1)}\, y^n \cdot \cancel{x} + y^{(n+1)} \cdot x + n \cdot y^n \cdot 1$
$+ y^n \cdot 1$

$\cancel{x} \cdot 1$

$y^n = y^{(n+2)} \cdot x^2 + 2xn y^{(n+1)} + n(n-1) y^{(n)} + x y^{(n+1)} + n y^n$
$+ y^n$

$y^n = x^2 y^{(n+2)} + 2nx y^{(n+1)} + (n^2 - n) y^{(n)} + x y^{(n+1)} + y^n + n y^n$

$y^n = x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^{(n)} (n^2 - n + 1 + n)$

$y^n = x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^{(n)} (n^2 + 1) = 0$

$x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^{(n)} (n^2 + 1) = 0$