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15/ENG06/010
Mechanical Engineering

1) If $y = e^{x^2 + x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x + 1$

$2x + 1e^u \qquad u = x^2 + x$

$\frac{dy}{dx} = 2x + 1e^{x^2 + x}$

$\frac{d^2y}{dx^2} = 2e^{x^2} + u + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{x^2} + x + 4x^2 + 4x + 1 + e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2}, \quad y' = \frac{dy}{dx}, \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y' = (2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

w_1

$u = y^1$ $\quad v = 1$

$u^n = y^{n+2}$ $\quad v = 0$

$= y^{n+2} \cdot 1 + 0$

w_2

$u = y^1$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v = 2$

$u^{n-1} = y^n$ $\quad v = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

w_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v = 0$

$= 2(y^n \cdot 1) + 0$

$= 2y^n$

$w_2 = w_2 + w_3$

$y^{n+1} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a) Using Leibnitz theorem given that

$y = x^3 e^{4x}$ determine $y^{(5)}$

Soln

$u = e^{4x}$ $\quad v = x^3$

$y^{(5)} = u^{(5)}v + 5u^4 v^1 + 10u^3 v^2 + 10u^2 v^3 + 5uv^4 + uv^5$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) + 5(4^2 e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} \cdot x + 480 e^{4x}$

b)

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$x^2y'' + xy' + y = 0$$

$\downarrow \quad \downarrow \quad \downarrow$

$w_1 \quad w_2 \quad w_3$

$$w_1 + w_2 + w_3 = 0$$

For w_1

$u = y''$ $\qquad v = x^2$

$u' = y^{n+2}$ $\qquad v' = 2x$

$u^{n-1} = y^{n+1}$ $\qquad v'' = 2$

$$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)}{2!}y^n \cdot x = 0$$

$$= x^2y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y.$$

For w_2

$u = y'$ $\qquad v = x$

$u' = y^{n+1}$ $\qquad v' = 1$

$u^{n-1} = y^n$ $\qquad v'' = 0$

$$= y^{n+1} \cdot x + ny^n + 0$$

For w_3

$u = y$ $\qquad v = 1$

$u' = y^n$ $\qquad v' = 0$

$u^{n-1} = y^{n-2}$

$$w_1 + w_2 + w_3 = 0$$

$$\Rightarrow x^2y^{n+2} + 2nx \cdot y^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n$$

$$\Rightarrow xy^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2y^n - ny^n + ny^n + y^n$$

$$= x^2y^{n+2} + 2n + 1(xy^{n+1}) + (n^2+1)y^n.$$