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15 / ENG 06 / 031

Mechanical Engr

ENG 381 - Engineering Maths III

① if $y = e^{x^2+x}$

Show that

$$y'' = y'(2x+1) + 2y$$

$$y = e^{x^2+x}$$

$$\frac{dy}{dx} = y'$$

let $y = e^u$, $\frac{dy}{dx} = y' = \frac{dy}{du} \times \frac{du}{dx}$ $u = x^2 + x$

$$\frac{dy}{dx} = y' = e^u \times 2x + 1$$

$$y' = (2x+1)e^u$$

$$y' = (2x+1)e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = y''$$

$$y' = (2x+1)e^{x^2+x}$$

Using product rule

$$\text{let } v = 2x+1$$

$$v = e^{x^2+x}$$

$$\frac{dv}{dx} = 2$$

$$\frac{dv}{dx} = 2x+1 e^{x^2+x}$$

$$\therefore \frac{d^2y}{dx^2} = y'' = v \frac{dv}{dx} + u \frac{dv}{dx}$$

$$= 2(e^{2x+2}) + 2x+1(2x+1)e^{2x+2}$$

Since $y = e^{2x+2}$

$$y' = (2x+1)e^{2x+2}$$

$$y'' = 2(y) + (2x+1)(y')$$

$$y'' = 2y + (2x+1)y'$$

$$y'' = y'(2x+1) + 2y$$

Have prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

$$y'' = y'(2x+1) + 2y$$

$$y'' - y'(2x+1) - 2y = 0$$

Using Leibnitz theorem

① y''

$$u = y'' \quad v = 1$$

$$u^n = y^{n+2} \quad v' = 0$$

$$= u^n v + n u^{n-1} v'$$

$$= y^{n+2} (1) + n (y^{(n+1)}) (0)$$

$$= y^{n+2}$$

②

$$-y'(2x+1)$$

$$u = y' \quad v = -(2x+1)$$

$$u^n = y^{n+1} \quad v' = -2$$

$$v'' = 0$$

$$= u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v''$$

$$= -y^{n+1} (2n+1) + n(y^n)(-2)$$

$$= -(2n+1)y^{n+1} - 2ny^n$$

(10)

$$-2y$$

$$u = y$$

$$v = 2$$

$$u' = y'$$

$$v' = 0$$

$$= u^n v + n u^{n-1} v'$$

$$= -2y^n$$

$$\therefore y^{n+2} = -(2n+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2n+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2n+1)y^{(n+1)} + 2y^{(n+1)}$$

$$y^{n+2} = (2n+1)y^{(n+1)} + 2(n+1)y^n$$

(2) Using the Leibnitz theorem given find

$$y = x^3 e^{4x}$$

determine $y^{(5)}$

$$v = x^3$$

$$u = e^{4x}$$

$$v' = 3x^2$$

$$u' = 4e^{4x}$$

$$v'' = 6x$$

$$u'' = 16e^{4x}$$

$$v''' = 6$$

$$u''' = 64e^{4x}$$

$$v^{(4)} = 0$$

$$u^{(4)} = 256e^{4x}$$

$$v^{(5)} = 0$$

$$u^{(5)} = 1024e^{4x}$$

$$y^5 = u^5 v + n u^4 v' + \frac{n(n-1)}{2!} u^3 v^2 + \frac{n(n-1)(n-2)}{3!} u^2 v^3$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} u v^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u v^5$$

$$y^5 = \left[1024 e^{4x} (x^3) \right] + \left[5(256 e^{4x}) 3x^2 \right] + \left[\frac{5 \times 4}{2} \times 64 e^{4x} \right]$$

$$\times 6x \left] + \left[\frac{5 \times 4 \times 3}{3!} \times 16 e^{4x} \times 6 \right] + [0][0]$$

$$y^5 = 1024 e^{4x} x^3 + 1280 e^{4x} (3x^2) + 640 e^{4x} (6x)$$

$$+ 160 e^{4x} (6)$$

$$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x}$$

$$+ 960 e^{4x}$$

$$y^5 = e^{4x} (1024 x^3 + 3840 x^2 + 3840 x + 960)$$

$$(1) \quad x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$(4) \quad x^2 y''$$

$$u = y''$$

$$u^n = y^{n+2}$$

$$v = x^2$$

$$v' = 2x$$

$$v'' = 2$$

$$v''' = 0$$

$$y^n = u^n u + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$$

$$y^n = y^{n+2} x^2 + n y^{n+1} (2x) + \frac{n(n-1)}{2!} y^n (2) + 0$$

$$y^n = y^{n+2} x^2 + 2n y^{n+1} + n(n-1) y^n$$

(2)

$$xy'$$

$$u = y'$$

$$u' = y''$$

$$v = x$$

$$v' = 1$$

$$v'' = 0$$

$$y^n = u^2 v + n v^2 - v' + \frac{n(n-1)}{2!} u^{n-2} v^4$$

$$y^n = y^{n+1} x + n y^n + 0$$

$$y^n = x y^{n+1} + n y^n$$

(3)

$$y$$

$$u = y$$

$$u' = y'$$

$$v = 1$$

$$v' = 0$$

$$y^n = u^n u + n u^{n-1} v'$$

$$= y^n (y') + n (y^{n-1}) (0)$$

~~$$y^n = y^{n+2} x^2 + n y^{n+1} 2x + (n(n-1)) y^n + x(y^{n+1}) + n y^2 + y^n$$~~
$$y^n = x^2 (y^{n+2}) + 2x n (y^{n+1}) + (n^2 - n) y^n + x(y^{n+1}) + n y^2 + y^n$$

$$y^n = (n^2 - n + n + 1) y^n + (2x n + x) y^{n+1} + x^2 (y^{n+2})$$

$$0 = x^2 (y^{n+2}) + (2x n + x) y^{n+1} + (n^2 + 1) y^n$$

$$0 = x^2 y (n+2) + (2n+1) x y^{n+1} + (n^2+1) y^n$$

$$x^2 y (n+2) + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$$