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15/ENG03/010
CIVIL ENGINEERING

1. $y = e^{x^2+x}$

$$\frac{dy}{dx} = (2x+1)\, e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = 2e^{x^2+\cancel{x}} + (2x+1)(2x+1)\, e^{x^2+x}$$

$$= e^{x^2+x}\,(2x+1)(2x+1)+2$$

$y'(2x+1)+y.$

$$(2x+1)\, e^{x^2+x}\,(2x+1) + 2y$$

$$(2x+1)\, e^{x^2+x}\,(2x+1) + 2e^{x^2+x}$$

$$= e^{x^2+x}\,(2x+1)(2x+1)+2$$

$$y^2 = y'(2x+1) + 2y$$

1st Product $u = y^2$; $u^n = y^{(n+2)}$; $u^{n-1} = y^{n+1}$; $u^{n-2} = y^n$

$v = 1$ $v' = 0$

2nd $u = y'$; $u^n = y^{(n+1)}$ $y^{n-1} = y^n$

$v = 2x+1$, $v' = 2$; $v^2 = 0$

3rd $u = y$, $u^n = y^n$

$v = 2$; $v' = 0$

1st Product = 2nd product + 3rd product

$$y^{n+2} = n(y^{n+1})2 + ny^{(n)}\cdot 2$$

$$y^{n+2} = y^{n+1}(2x+1) + ny^n\cdot 2 + y^n 2.$$

$$= y^{n+1}(2x+1) + 2ny^n + 2y^n$$

$$= (2x+1)y^{n+1} + 2(n+1)y^n,$$

2. $y = x^3 e^{4x}$

$u = e^{4x}$ $\quad$ $v = x^3$

$u^n = 4^n e^{4x}$ $\quad$ $v' = 3x^2$

$u^{n-1} = 4^{(n-1)} e^{4x}$ $\quad$ $v^2 = 6x$

$u^{n-2} = 4^{(n-2)} e^{4x}$ $\quad$ $v^3 = 6$

$u^{n-3} = 4^{(n-3)} e^{4x}$ $\quad$ $v^4 = 0$

$= 4^n e^{4x} x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!}$

$4^{n-3} e^{4x} \cdot 6$

$n = 5$

$= 4^5 e^{4x} x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + \frac{10 \cdot 4^3 e^{4x} \cdot 6x}{2 \times 1} + 10 \cdot 4^2 e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 1920 e^{4x} + 160 e^{4x}$

3. $\cancel{x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + y = 0}$ $\quad$ Show that $x^2 y^{(n+2)} + 2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$

$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y^{(2)} + x y^{(1)} + y = 0$

1 $\quad$ 2 $\quad$ 3

1st product $\quad$ $u = y^2$; $u^n = y^{(n+2)}$; $u^{n-1} = y^{(n+1)}$; $u^{n-2} = y^{(n)}$

$v = x^2$; $v' = 2x$; $v^2 = 2$; $v^3 = 0$

2nd Product

$u = y'$; $u^n = y^{n+1}$; $u^{n-1} = y^n$

$v = x$; $v' = 1$; $v'' = 0$

3rd product

$u = y$; $u^n = y^n$

$v = 1$; $v' = 0$

$= y^{n+2} \cdot x^2 + n y^{n+1} \cdot 2x + \frac{n(n-1)}{2!} y^n \cdot 2 + y^{n+1} \cdot x + n y^n + y^n = 0$

$x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1) y^n + x y^{n+1} + n y^n + y^n = 0$

$x^2 y^{n+2} + 2xn y^{(n+1)} + x y^{(n+1)} + n(n-1) y^n + n y^n + y^n$

$= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^n (n(n-1) + n + 1)$

$= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^n (n^2 \cancel{- n + n} + 1)$

$= x^2 y^{n+2} + x y^{(n+1)} (2n+1) + y^n (n^2+1) = 0$