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15/ENG06/067.
Mechanical Engineering

(1) $y = e^{x^2+x}$.

Solution.

$y' = (2x+1)\, e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1) \cdot e^{x^2+x}$

$y'' = 2y + (2x+1)y'$.

$\therefore$

$y'' = y'(2x+1) + 2y$

$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + (n+1)\, y^n \cdot 2$

$y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2(n+1)\, y^n$

2i $y = x^3 e^{4x}$

$$y^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} v^3 + \cdots +$$

let $x^3 = v$ and $u = e^{4x}$.

$y^5 = (4)^5 e^{4x} \cdot x^3 + 5(4)^4 e^{4x} + 10(4)^3 e^{4x} \cdot 6x + 10(4)^2 e^{4x} \cdot 6 + 0$.

$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x} + 0$.

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$.

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

$x^2 y'' + xy' + y^0 = 0$.

$x^2 y^{(2)} + xy^{(1)} + y^0 = 0$.

Using leibnitz theorem.

$y^n = y^{(n+2)} \cdot x^2 + n \cdot 2xy^{(n+1)} + \frac{2n \cdot (n-1)}{2!} \cdot y^n + y^{(n+1)} \cdot x + n \cdot y^{(n)} \cdot 1 + y^n$

$y^n = x^2 y^{(n+2)} + 2xy^{(n+1)} \cdot n + n(n-1)\, y^n \frac{2!}{2!} + xy^{(n+1)} + ny^{(n)} + y^{(n)}$

$y^n = y^{(n+2)}(x^2) + y^{(n+1)}(2xn + x) + y^{(n)}(n(n-1) + n + 1)$.

$y^n = x^2 y^{(n+2)} + (2n+1)\, xy^{(n+1)} + (n^2 - n + n + 1)\, y^{(n)}$.

$y^n = x^2 y^{(n+2)} + (2n+1)\, xy^{(n+1)} + (n^2-1)\, y^{(n)}$.

equate to 0.

$x^2 y^{(n+2)} + (2n+1)\, xy^{(n+1)} + (n^2-1)\, y^{(n)} = 0$.