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Q1.

$$y = e^{x^2+x}$$

$$\frac{dy}{dx} = (2x+1)$$

$$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$= e^{x^2+x} (2x+1)(2x+1) + 2$$

$$y'(2x+1) + y$$

$$(2x+1)e^{x^2+x}(2x+1) + 2e^{x^2+x}$$

$$(2x+1)e^{x^2+x}(2x+1) + 2e^{x^2+x}$$

$$= e^{x^2+x} \{ (2x+1)(2x+1) + 2 \}$$

$$y'' = y'(2x+1) + 2y$$

$$y' = w_2$$

$$u = y' \quad v = 2x+1$$

$$u^A = y^{n+1} \quad v' = 2$$

$$u^{n-1} = y^n \quad v'' = 0$$

$$w_1 = u = y^2$$

$$u^n = y^{(n+2)}$$

$$w_3 = u = y \quad v = 2$$

$$u^n = y^n \quad v = 0$$

$$w_1 = w_2 + w_3$$

$$\text{or } w_1 + w_2 + w_3 = 0$$

$$y^{(n+2)} = n(y^{n+1}) \cdot 2 + ny^{(n)} \cdot 2$$

$$y^{n+2} = y^{n+1} (2n+1) + ny^{n+1} + y^{n+1} \cdot 2$$

$$= y^{(n+1)} (2n+1) + 2ny^n + 2y^n$$

$$= (2n+1)y^{(n+1)} + 2(n+1)y^n$$

(2)  $y = x^3 e^{4x}$

$$u = e^{4x}$$

$$v = x^3$$

$$u^n = 4^n e^{4x}$$

$$v' = 3x^2$$

$$u^{n-1} = 4^{(n-1)} e^{4x}$$

$$v^2 = 6x$$

$$u^{n-2} = 4^{(n-2)} e^{4x}$$

$$v^3 = 6$$

$$u^{n-3} = 4^{(n-3)} e^{4x}$$

$$v^4 = 0$$

$$= 4^n e^{4x} \cdot 3 + n \cdot 4^{(n-1)} e^{4x} \cdot 3x^2 + n(n-1) \cdot 4^{(n-1)} e^{4x} \cdot 6x + n(n-1)(n-2) \cdot 4^{(n-2)} e^{4x} \cdot 6$$

$$n = 5$$

$$3!$$

$$15 e^{4x} \cdot 3 + 5 \cdot 4 e^{4x} \cdot 3x^2 + 10 \cdot 4 e^{4x} \cdot 6x + 10 \cdot 4 e^{4x} \cdot 6$$

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

Show that  $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

$$x^2 y^{(2)} + x y^{(1)} + y = 0$$

$w_1$        $w_2$        $w_3$

$w_1$	
$u = y^{(2)}$	$v = x^n$
$u^{(n)} = y^{(n+2)}$	$v' = 2x$
$u^{(n-1)} = y^{(n+1)}$	$v'' = 2$
$u^{(n-2)} = y^{(n)}$	

$w_2$	
$u = y^{(1)}$	$v = x$
$u^{(n)} = y^{(n+1)}$	$v' = 1$
$u^{(n-1)} = y^{(n)}$	$v'' = 0$

$w_3$

$$u = y$$

$$u^{(n)} = y^{(n)}$$

$w_1 + w_2 + w_3$

$$x^2 y^{(n+2)} + nxy^{(n+1)} + n(n-1)y^n + 2xy^{(n+1)} + ny^n + y^n = 0$$

2!

$$\begin{aligned}
&= x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0 \\
&= x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n \\
&= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n(n-1) + n + 1) \\
&= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) \\
&= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 + 1) = 0
\end{aligned}$$