

Allen-Operator Kingsley
Civil Engineering

(5/ENG03/006)

ENGA 381 Assignment III

$$1) y = e^{x^2+x}$$
$$\text{let } u = x^2+x$$
$$\frac{dy}{dx} = 2x+1$$

$$y = e^u$$
$$\frac{dy}{du} = e^u$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$\frac{dy}{dx} = e^u \cdot 2x+1$$

Where $u = x^2+x$

$$\frac{dy}{dx} = e^{x^2+x} \cdot (2x+1) = y'$$

if $y' = e^{x^2+x} (2x+1)$

then $y'' = 2e^{x^2+x} (2x+1) + [e^{x^2+x} \cdot (2x+1)] (2x+1)$

$$y'' = 2e^{x^2+x} (2x+1) + e^{x^2+x} (2x+1) \cdot (2x+1)$$

but $y = e^{x^2+x} (2x+1)$

$$\text{and } y' = e^{x^2+x} (2x+1)$$

$$y'' = 2(y) + y' (2x+1)$$

Applying Leibnitz theorem to the above equation find the n^{th} derivative

$$y^{(n+2)} = 2(n+1)y'' + y^{(n+1)}(2x+1)$$

2) $y = x^3 e^{4x}$ determine $y^{(5)}$

Solution

$$y^{(5)} = u^{(5)}v + 5u^{(4)}v' + 10u^{(3)}v'' + 10u^2v''' + 5u^4v^{(4)} + v^{(5)}$$

Where $u = e^{4x}$

$$u' = 4e^{4x}$$

$$u'' = 16e^{4x}$$

$$u''' = 64e^{4x}$$

$$u^{(4)} = 256e^{4x}$$

$$u^{(5)} = 1024e^{4x}$$

$$v = x^3$$

$$v' = 3x^2$$

$$v'' = 6x$$

$$v''' = 6$$

Therefore

$$y^{(5)} = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x})(6) + 5(4e^{4x})(0) + 0^5$$

$$= 1024x^3 \cdot e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}$$

$$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

a) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

Show that $x^{2n} y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^{(n)} = 0$

Solution

The equation can be written as $x^2 y'' + x y' + y = 0$

Let $w_1 = x^2 y''$, $w_2 = x y'$ and $w_3 = y$

Solving for $w_1 = x^2 y''$

Let $u = y^2$; $u' = y^{n+2}$

Let $v = x^2$; $v' = 2x$ $v'' = 2$ $v''' = 0$

$$w_1 = y^{(n+2)}(x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2!} y^{(n)} 2 + 0)$$

$w_2 = x y'$

Let $u = y'$; $u' = y''$

Let $v = x$; $v' = 1$ and $v'' = 0$

$$w_2 = y^{(n+1)}(x + n y^{(n)} \cdot 1 + 0)$$

$$= x y^{(n+1)} + n y^n$$

$w_3 = y^{(n)}$

Combining

$$W = w_1 + w_2 + w_3$$

$$\begin{aligned}
 W &= x^2 y^{(n+2)} + \beta y^{(n+1)} - 2x + \frac{n(n-1)}{2} y^{(n+2)} + 2xy' + y^{(n+1)} x + ny'' + y^{(n)} = 0 \\
 &= x^2 y^{(n+2)} + (x y^{(n+1)}) (2n+1) + y^{(n)} [n(n-1) + n+1] = 0 \\
 &= x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0
 \end{aligned}$$