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15/ENG04/036
course : ENG 381

If $y = e^{x^2 + x}$

$y' = (2x+1) e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x}$

$y'' = y'(2x+1) + 2y$

$y' = (2x+1) e^{x^2+x}$

$y'(2x+1) = (2x+1) e^{x^2+x} (2x+1)$

$= (2x+1)^2 e^{x^2+x}$

$2y = 2 \cdot e^{x^2+x}$

$\therefore y'(2x+1) + 2y = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$2e^{x^2+x} + (2x+1)^2 e^{x^2+x} = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$u = y''$

$u^n = y^{n+2}$

$P = y'(2x+1)$

$v' = 2$

$v'' = 0$

$u = y'$

$u^n = y^{n+1}$

$p^n = y^{n+1} \cdot 2x+1 + n \cdot y^n \cdot 2$

$S = 2y$

$S^n = 2y^n$

$u^n = P^n + 5$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n-2} = (2x+1)y^{n+1} + 2(n+1)y^n$

Question 2:

Using the theorem, given that

$y = x^3 e^{4x}$ determine y^5

$v = x^3$ $u = e^{4x}$

$v' = 3x^2$ $u' = 4e^{4x}$

$v'' = 6x$ $u'' = 4 \cdot 4e^{4x}$

$v''' = 6$ $u''' = 4 \cdot 4 \cdot 4e^{4x}$

$v^{iv} = 0$ $u^{iv} = 4^n e^{4x}$

$$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{6} 4^{n-3} e^{4x} \cdot 6$$

$$y^n = 4^n e^{4x} x^3 + n4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^5 = 4^5 e^{4x} x^3 + 4e^{4x} 3x^2 + 5 \cdot 4 \cdot 4^3 e^{4x} 3x + 5 \cdot 4 \cdot 3 \cdot 4^2 e^{4x}$$

Question 2

$x^2y'' + xy' + y = 0$ show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)}$
$+ (n^2+1)y^n = 0$

let $w = x^2y''$

$V = x^2$ $\quad$ $U = y''$

$V' = 2x$ $\quad$ $U' = y'''$

$V'' = 2$ $\quad$ $U' = y^{(IV)}$

$V''' = 0$ $\quad$ $U^n = y^{n+2}$

$W^n = y^{n+2}x^2 + n \cdot y^{n+1} 2x + n\frac{(n-1)}{2} \cdot y^n$

$W^n = x^2y^{n+2} + n2xy^{n+1} + n(n-1)y^n$

let $p = xy'$

$V = x$ $\quad$ $U = y'$

$V' = 1$ $\quad$ $U' = y''$

$V'' = 0$ $\quad$ $U^n = y^{n+1}$

$p^n = y^{n+1}x + n \cdot y^n \cdot 1$

$p^n = xy^{n+1} + ny^n$

$S = y$

$S^n = y^n$

$y^{n+2} \cdot x^2 + n \cdot 2xy^{n+1} + n(n-1)y^n + y^{n+1}x + ny^n + y^n = 0$

$y^{n+2}x^2 + (n \cdot 2xy^{n+1} + xy^{n+1}) + [n(n-1)y^n + ny^n + y^n] = 0$

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$$x^2 y^{n+2} + (2n+1)xy^{n+1} + [n(n-1) + 1 + n]y^n = 0$$

$$x^2 y^{n+2} + (2n+1)\, xy^{n+1} + (n^2+1)y^n = 0$$