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ASSIGNMENT 3

(i) $y = e^{x^2+x}$

Using Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$u = x^2 + x$ $\qquad$ $y = e^u$

$\frac{du}{dx} = 2x + 1$ $\qquad$ $\frac{dy}{du} = e^u$

$$\frac{dy}{dx} = e^u \times (2x+1) = (2x+1)e^u$$

$$\frac{dy}{dx} = (2x+1)e^{x^2+x} \implies y' = (2x+1)e^{x^2+x}$$

Since $y = e^{x^2+x}$ and
$dy/dx = (2x+1)e^{x^2+x}$

To find $y'' \Rightarrow d^2y/dx^2$, we use product rule

$\Rightarrow udv + vdu$

$u = e^{x^2+x}$ $\qquad$ $v = 2x+1$

$du = (2x+1)e^{x^2+x}$ $\qquad$ $dv = 2$

$$\frac{d^2y}{dx^2} = e^{x^2+x}(2) + (2x+1)(2x+1)e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$$

Recall: $y = e^{x^2+x}$ and $y' = (2x+1)e^{x^2+x}$

$$\therefore y'' = y'(2x+1) + 2y$$

$$\underbrace{y''}_{W_1} = \underbrace{y'(2x+1)}_{W_2} + \underbrace{2y}_{W_3}$$

W_1:
$u = y^{(2)}$
$u^{(n)} = y^{(n+2)}$

W_2:
$u = y^{(1)}$ $\qquad$ $V = 2x+1$
$u^{(n)} = y^{(n+1)}$ $\qquad$ $V^{(1)} = 2$

W_3:
$u = y$ $\qquad$ $V = 2$
$u^{(n)} = y^n$ $\qquad$ $V^{(1)} = 0$

$w_1 = w_2 + w_3$

$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^n 2 + y^n 2$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$

$\therefore y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

(2)(i) $y = x^3 e^{4x}$

$u = e^{4x}$	$V = x^3$
$u^{(n)} = 4^n e^{4x}$	$V^{(0)} = x^3$
$u^{(n-1)} = 4^{n-1} e^{4x}$	$V^{(1)} = 3x^2$
$u^{(n-2)} = 4^{n-2} e^{4x}$	$V^{(2)} = 6x$
$u^{(n-3)} = 4^{n-3} e^{4x}$	$V^{(3)} = 6$
$u^{(n-4)} = 4^{n-4} e^{4x}$	$V^{(4)} = 0$

$$y^{(n)} = u^{(n)}V^{(0)} + nu^{(n-1)}V^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}V^{(2)} + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}V^{(3)} + \frac{n(n-1)(n-2)(n-3)}{4!}u^{(n-4)}V^{(4)}$$

$$y^{(n)} = 4^n e^{4x}.x^3 + n4^{n-1}e^{4x}.3x^2 + \frac{n(n-1)}{2!}4^{n-2}e^{4x}.6x + \frac{n(n-1)(n-2)}{3!}4^{n-3}e^{4x}.6 + \frac{n(n-1)(n-2)(n-3)}{4!}4^{n-4}e^{4x}.0$$

$$y^{(n)} = 4^n e^{4x}.x^3 + n4^{n-1}e^{4x}.3x^2 + n(n-1)4^{n-2}e^{4x}.3x + n(n-1)(n-2)4^{n-3}e^{4x}$$

$$y^{(n)} = e^{4x}[4^n x^3 + n4^{n-1}3x^2 + n(n-1)4^{n-2}3x + n(n-1)(n-2)4^{n-3}]$$

when $n=5$

$$y^{(5)} = e^{4x}[(4^5)(x^3) + (5)(4^4)(3x^2) + (5)(4)(4^3)(3x) + (5)(4)(3)(4^2)]$$

$$y^{(5)} = e^{4x}[1024x^3 + 3840x^2 + 3840x + 960]$$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$\Rightarrow x^2 y'' + xy' + y = 0$

$\Rightarrow x^2 y^{(2)} + xy^{(1)} + y = 0$

$w_1 = x^2 y^{(2)}$

$u = y^{(2)}$ $\qquad\qquad$ $v = x^2$

$u^{(n)} = y^{(n+2)}$ $\qquad\qquad$ $v^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\qquad\qquad$ $v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\qquad\qquad$ $v^{(3)} = 0$

$$w_1^{(n)} = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)y^{(n)}\cancel{2}}{\cancel{2!}}$$

$$w_2^{(n)} = y^{(n+1)}x + ny^{(n)}$$

$$w_3^{(n)} = y^{(n)}$$

$\Rightarrow x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$

$\Rightarrow x^2y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)} = 0$

$\Rightarrow x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n(n-1) + n + 1) = 0$

$\Rightarrow x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2 - \cancel{n} + \cancel{n} + 1) = 0$

$\Rightarrow x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$ //