Dominic Turpoint Dominic
15/ENG/0021
Mechanical Engineering

ENG 381

1) $y = e^{2x} + 2x$

$\frac{dy}{dx} = 2x + 1$

$y = e^{ax}$

$\frac{dy}{du} = e^{u}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^{u} \times 2x + 1$

$2x + 1 e^{u} \quad u = x^2 + x$

$\frac{dy}{dx} = 2x + 1 e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1 e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1 + e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \quad y' = \frac{dy}{dx} \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1 e^{x^2+x}$

$y' = (2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1 e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y' = (2x+1) \cdot 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$\downarrow$ $\quad$ $\swarrow$ $\quad$ $\downarrow$

$Vol_1 \quad Vol_2 \quad Vol_3$

$u = y'' \qquad u = 1$

$v^n = y^{n+2} \qquad v = 0$

$= y^{n+2} \cdot 1 + 0$

$u = y'$

$u^n = y^{n+1}$

$v^{n-1} = y^n$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$y^{n+1}(2x+1) + 2n(y^n)$

Vol_3

$u = y \qquad v = 1$

$u^n = y^n \qquad v = 0$

$= 2(y^n - 1) + 0$

$Vol_1 = Vol_2 + Vol_3$

$$y^{n+1} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$$
$$= y^{n+1}(2x+1) + 2(n+1)y^n$$

2) $y = x^3 e^{4x}$

$u = e^{4x} \quad v = x^3$

$$y^5 = \frac{u^5}{5}v + 5u^4u' + 10u^2u'' + 10u^2v^3 + 5u'v^4 + u^5$$

$$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) + 5(4^2 e^{4x} \cdot 6) + 0$$

$$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$$

$$= 1024 e^{4x} + 3840 x^2 + 3840 e^{4x} x + 480 e^{4x}$$

3) $\frac{x^2 dy^2}{dx^2} + \frac{x dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$val_1 \quad val_2 \quad val_3$

$val_1 + val_2 + val_3 = 0$

For val_1

$u = y'' \quad v = x^2$

$u^n = y^{n+2} \quad v' = 2x$

$u^{n-1} = y^{n+1} \quad v'' = 2$

$= y(n+2)(x^2) + n(yn+1)2x + \frac{n(n-1)y^n \cdot x}{2!} = 0$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

For val_2

$u = y'$ $\quad u = x$

$u^n = y^{n+1}$ $\quad u = 1$

$u^{n-1} = y^n$ $\quad u^n = 0$

$= y^{n+1} \cdot x + ny^n + 0$

For val_3

$u = y$ $\quad u = 1$

$u^n + y^n$ $\quad v^1 = 0$

$= y^n \cdot 1$

$val_1 + val_2 + val_3 = 0$

$x^2 y^n + 2 + 2nxy^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n$

$xy^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2 y^n - ny^n + ny^n + y^n$

$x^2 y^{n+2} + 2n+1(xy^{n+1}) + (n^2+1)y^n$