Omajugho mobe leslie
16/ENG06/086

(1) IF $y = e^{x^2+x}$

IF $(x^2+x) = u$

$\frac{dy}{dx} = 2x+1$

$\frac{dy}{dx} = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = (4x^2+4x+1)e^{x^2+x}$

IF

$y'' = y'(2x+1) + 2y$

$(4x^2+4x+1) = (2x+1)e^{x^2+x} + 2(e^{x^2+x})$

$(4x^2+4x+1)e^{x^2+x} = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$

$= -y'' + y'(2x+1) + 2y$

module I

$w = -y''$

$w' = -y'''$

$w'' = -y^{iv}$

$w^n = -y^{n+2}$

module 2

$y'(2x+1)$

$u = y'$ $\quad$ $v = 2x+1$

$u' = y''$ $\quad$ $v' = 2$

$u'' = y'''$ $\quad$ $v'' = 0$

$u^n = y^{n+1}$

Using lewis equation

$w^n = {}^nC_0 u^n v^0 + {}^nC_1 u^{n-1}v' + {}^nC_2 u^{n-2}v^2$

$y^{n+1}(2x+1) + ny^n 2 + 0$

$B = y^{n+1}(2x+1) + 2ny^n$

For module 3

$2y$

let $M = 2y$

$m^n = 2y^n$

$w^n + B^n + m^n \Rightarrow -y^{n+2} + y^{n+1}(2x+1) + 2ny^n + 2y^n$

$-y^{n+2} + y^{n+1}(2x+1) + 2ny^n + 2y^n$

$-y^{n+2} + y^{n+1}(2x+1) + 2(n+1)y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

(2) Using the Leibnitz theorem given that

$y = x^3e^{4x}$ determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u' = 4e^{4x}$ $\quad v' = 3x^2$

$u'' = 16e^{4x}$ $\quad v'' = 6x$

$u''' = 64e^{4x}$ $\quad v''' = 6$

$u^{iv} = 256e^{4x}$ $\quad v^{iv} = 0$

$u^v = 1024e^{4x}$ $\quad 0$

$u^n v^0 + {}^nC_1 u^{n-1}v^1 + {}^nC_2 u^{n-2}v^2 + {}^nC_3 u^{n-3}v^3 + {}^nC_4 u^{n-4}v^4 + {}^nC_5 u^{n-5}v^5$

$u^n v^0 + nu^{n-1}v^1 + \frac{n(n-1)}{2}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^3 + \frac{n(n-1)(n-2)(n-3)}{4!}u^{n-4}v^4 +$

$+\frac{(n-1)(n-2)(n-3)(n-4)}{5!}u^{n-5}v^5$

y^5

$y^n = n = 5$

$u^5v^0 + 5u^{(5-1)}v^1 + \frac{5(5-1)}{2}u^{(5-2)}v^2 + \frac{5(5-1)(5-2)}{6}u^{(5-3)}v^3 + \frac{5(5-1)(5-2)(5-3)}{24}u^{5-4}v^4$

$\frac{(5-1)(5-2)(5-3)(5-4)}{120}u^{5-5}v^5$

$[1024e^{4x} \cdot x^3] + [5 \times 256e^{4x} 3x^2] + [\frac{5\times4}{2} \cdot 64e^{4x}6x] + \frac{5\times4\times3}{6} \cdot 16e^{4x} \cdot 6 + 0 + 0$

$1024x^3e^{4x} + 3840x^2e^{4x} + 3600xe^{4x} + 960e^{4x}$

(11) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$x^2y'' + xy' + y = 0$

module 1

x^2y''

$u = y'' \quad v = x^2$

$u' = y''' \quad v' = 2x$

$u'' = y^{iv} \quad v'' = 2$

$u''' = y^{v} \quad v''' = 0$

$u^n = y^{n+2}$

${}^nC_0 u^n v^0 + {}^nC_1 u^{n-1}v^1 + {}^nC_2 u^{n-2}v^2 + {}^nC_3 u^0 v^3$

$y^{n+2}x^2 + ny^{n+1} \cdot 2x + \frac{n(n-1)}{2}y^n \cdot 2 + 0$

$x^2y^{n+2} + 2xny^{n+1} + \frac{2(n-1)}{2}y^n$

$x^2y^{n+2} + 2xny^{n+1} + n(n-1)y^n$

module 2

$= xy'$

$u = y' \quad v = x$

$u' = y'' \quad v' = 0$

$u^n = y^{n+1}$

${}^nC_0 u^n v^0 + {}^nC_1 u^{n-1}v^1$

$y^{n+1} \cdot x + 0$

$u^{5-4}v^4$

xy^{n+1}

module 3

$y = y^n$

$=$

$x^2y^{n+2} + 2xny^{n+1} + n(n-1)y^n + xy^{n+1} + y^n$

$x^2y^{n+2} + (2n+1)xy^{n+1} + n(n-1)y^n + y^n$

$x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2-n)y^n + y^n$