Ekanola Adeyaniy Oluwapelumi
16/ENG04/065
Elect/Elect
ENG381

1.) If $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$

and hence, prove that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$y = e^{x^2+x}$

$y' = (2x-1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

$y'' = y'(2x+1) + 2y$

$y^{(n+2)} = y^{(n+1)}(2x+1) + (n+1)y^{(n2)}$

$\therefore\ y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

2.) Using the Leibnitz theorem, given that

i.) $y = x^3e^{4x}$, determine y^5

ii.) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$, show that

$x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$

Soln

i.) $y = x^3e^{4x}$

$y^n = u^nv + nu^{n-1}v' + \dfrac{n(n-1)}{2!}u^{n-2}v^2 + \ldots$

let $x^3 = v$ and $e^{4x} = u$

$y^5 = 4^5 \cdot e^{4x} \cdot x^3 + 5 \cdot 4^4 \cdot e^{4x} \cdot 3x^2 + 10 \cdot 4^3 \cdot e^{4x} \cdot 6x + 10 \cdot 4^2 \cdot e^{4x} \cdot 6 + 0$

$= 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x} + 0$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

ii.) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y^0 = 0$

$x^2y^2 + xy' + y^0 = 0$

using Leibnitz theorem;

$y^n = y^{n+2}\cdot x^2 + n\cdot 2xy^{n+1} + \frac{2n(n-1)}{2!}y^n + y^{n+1}\cdot x + n\cdot y^n\cdot 1 + y^n$

$y^n = x^2y(n+2) + 2xy^{(n+1)}\cdot n + n(n-1)y^n + xy^{n+1} + ny^n + y^n$

$y^n = y^{n+2}\cdot x^2 + y^{n+1}(2xn + x) + y^n(n(n-1) + n + 1)$

$y^n = x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2 - n + n + 1)y^n$

$y^n = x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2-1)y^n$

$x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2-1)y^n = 0$