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Computer Engineering
300L

1) If $y = e^{x^2+x}$ Show that $y'' = y'(2x+1) + 2y$ and hence, prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln.

$y = e^{x^2+x}$; $u = e^{x^2+x}$, $v = 1$

$u^n = (2x+1)^n e^{x^2+x}$ $\quad v' = 0$

$u^{n-1} = (2x+1)^{n-1} e^{x^2+x}$

$y^{(n)} = u^n v^0 + n u^{(n-1)} v^{(1)}$; $\quad y^n = (2x+1)^n e^{x^2+x}$

let $n=1$

$y' = (2x+1)' e^{x^2+x}$; $\quad u = e^{x^2+x}$; $\quad v = 2x+1$

$u^n = (2x+1)^n e^{x^2+x}$ $\quad v' = 2$

$u^{n-1} = (2x+1)^{n-1} e^{x^2+x}$ $\quad v'' = 0$

$y'^{(n)} = (2x+1)^n e^{x^2+x} \cdot (2x+1) + n(2x+1)^{n-1} e^{x^2+x} \cdot 2$

$= (2x+1)(2x+1)^n e^{x^2+x} + 2n(2x+1)^{n-1} e^{x^2+x}$

let $n=1$

$y'' = (2x+1)(2x+1)' e^{x^2+x} + 2(2x+1)^0 e^{x^2+x}$

$y'' = (2x+1)(2x+1)' e^{x^2+x} + 2(e^{x^2+x})$

Substitute $y = e^{x^2+x}$ and $y' = (2x+1)' e^{x^2+x}$ in y''

$y'' = (2x+1)y' + 2y$ $\quad u = y,\ v = 2x+1,\ u = y,\ v = 2$

$y'' = y'(2x+1) + 2y$ $\quad u^n = y^{n+1},\ v' = 2 \quad u^n = y^n,\ v' = 0$

This can be written as $\quad u^{n-1} = y^n \quad v'' = 0$

$y^{(2)} = y^{(1)}(2x+1)2y$

$y^{(n)} = u^{(n)} v + n u^{(n-1)} v^{(1)}$

$y^{(n+2)} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

Using the Leibnitz theorem given that

$y = x^3 e^{4x}$, determine y^5

$u = e^{4x}$ $\qquad$ $v = x^3$

$u^n = 4^n e^{4x}$ $\qquad$ $v' = 3x^2$

$u^{n-1} = 4^{(n-1)} e^{4x}$ $\qquad$ $v'' = 6x$

$u^{n-2} = 4^{(n-2)} e^{4x}$ $\qquad$ $v''' = 6$

$u^{n-3} = 4^{(n-3)} e^{4x}$ $\qquad$ $v^{iv} = 0$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1) u^{(n-2)}}{2!} v'' + \frac{n(n-1)(n-2) u^{(n-3)}}{3!} v'''$$

$$y^n = 4^n e^{4x} \cdot x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1) 4^{(n-2)}}{2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2) \cdot 4^{(n-3)} e^{4x}}{3 \times 2} \cdot 6$$

$$y^n = 4^n e^{4x} \cdot x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + n(n-1) 4^{(n-2)} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{(n-3)} e^{4x}$$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{(5-1)} e^{4x} \cdot 3x^2 + 5(5-1) \cdot 4^{(5-2)} e^{4x} \cdot 3x + 5(5-1)(5-2) 4^{(5-3)} e^{4x}$$

$$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$$

$$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$$

ii $x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$ Show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)$

$= x^2 y'' + xy' + y = 0 \Rightarrow x^2 y^{(2)} + xy^{(1)} + y^{(0)} = 0$

$u = y''$	$v = x^2$	$u = y'$	$v = x$	$u = y$	$v = 1$
$u^n = y^{(n+2)}$	$v' = 2x$	$u^n = y^{n+1}$	$v' = 1$	$u^n = y^n$	$v' = 0$

$u^{(n-1)} = y^{(n+1)}$ $v'' = 2$ | $u^{n-1} = y^{n}$ $v'' = 0$ | ~~u =~~

$u^{(n+2)} = y^{n}$ $v'' = 0$ | ~~u~~

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1) u^{(n-2)}}{2!} v''$$

$$y^n = y^{n+2} x^2 + n y^{(n+1)} \cdot 2x + \frac{n(n-1) y^n \cdot 2}{2} + y^{n+1} \cdot x + n y^n \cdot 1 + y^n \cdot 1$$

$$\cancel{y^n = x^2 y^{(n+2)} + 2x\, n y^{(n+1)}}$$

$$y^n = x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n$$

$$y^n = x^2 y^{(n+2)} + 2nx y^{(n+1)} + x y^{(n+1)} + (n^2 - n) y^n + n y^n + y^n$$

$$y^n = x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 - n) + (n-1) y^n$$

$$y^n = x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$