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15/ENG04/003

ELECTRICAL ELECTRONICS

ENG 381

Assignment

1) Question $y = e^{x^2} + x$

Show that $y'' = y'(2x+1) + 2y$ and prove that $y(n+2) = (2x+1)y(n+1) + 2(n+1)y^n$

Solution

$y = e^{x^2} + x$ —— 1

$y' = (2x+1)e^{x^2+x}$ —— 2

$y^n = c_1^n e^{ax}$

where $\frac{udv}{dx} + \frac{Vdu}{dx} = \frac{dy}{dx}$

$u = 2x+1$; $du/dx = 2$

$v = e^{x^2+x}$; $dv/dx = (2x+1)e^{x^2+x}$

~~From~~ $y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

From eqn (1) and (2)

$y'' - y'(2x+1) + 2y$

let $w_1 = y'$ · $w = y'(2x+1)$

$u = y^2$

$u^n = y(2+n)$

$u = y(1)$ $\quad v = 2x+1$

$u^2 = y(1+n)$ $\quad v = 2$

$\cancel{y^{n+1} = y^n}$

$y^{n+1} = y^n$

$w_2 = 2y$

$u = y$ $\quad v = 2$

$u^n = y^{(n)}$

$w_1^{(n)} = w_2^{(n)} + w_3^{(n)}$

$y^n = u^{(n)}v + nu^{(n-1)}v''$

$y^{(2+n)} = y^{(1+n)} \cdot (2x+1) + n(y^{(n)}) \cdot 2 + y^{(n)} 2$

$y^{(2+n)} = (2x+1) y^{(1+n)} + 2(n+1) y^{(n)}$

2.) $y = x^3 e^{4x}$; determine $y^{(5)}$

Solution

$u = e^{4x}$

$u^n = 4^n e^{4x}$

$u^{n-1} = 4^{n-1} e^{4x}$

$u^{n-2} = 4^{n-2} e^{4x}$

$u^{n-3} = 4^{n-3} e^{4x}$

$$y^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1) u^{(n-2)} v''}{2!} + \frac{n(n-1)(n-2) u^{(n-3)} v}{3!}$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x}) \cdot 3x^2 + \frac{5(4)(4^3 e^{4x}) \cdot 6x}{2!} + \frac{5(4)(3)(4^2 e^{4x}) \cdot 6}{3!} + \dots$$

$y^{(5)} = 1024x^3 + 3840x^2 e^{4x} + 3840x e^{4x} + 96e^{4x}$

$y^{(5)} = 64e^{4x} [16x^3 + 60x^2 + 60x + 15]$

11) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Such that $n^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + (n^2+1) y^n = 0$

Solution

Let $w = x^2 y$

$u = y''$, $u' = y'''$, $u'' = y^{iv}$ .... $u^{(n)} = y^{(n+2)}$

$V = x^2$

$V' = 2x$

$V'' = 2$

$V''' = 0$

$$w^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1) u^{n-2} v''}{2!} + n(n-1)(n-2) u^{n-3} v'''$$

$$w^n = y^{n+2}x^2 + n \cdot y^{n+1} \cdot 2x + \frac{n(n-1)y^n \cdot 2 + 0}{2}$$

$$w^n = y^{n+2} \cdot x^n + n \cdot 2xy^{n+1} + n(n-1)y^n$$

Let $w = xy'$

$u = y'$; $u' = y''$; $u^n = y^{(n+1)}$

$v = x$

$v' = 1$

$v'' = 0$

$$w^{(n)} = u^n v^0 + \frac{nu^{n-1}v'}{1} + \frac{n(n-1)u^{n-2}v''}{2!} + \ldots$$

$$w^{(n)} = y^{n+1} + x + ny^n \cdot 1 + \frac{n(n-1)y^{n-1} \cdot 0}{2!}$$

$$w^{(n)} = y^{n+1} + x + ny^n$$

where $w = y$

$w^2 = y^n$

$\therefore y^{n+2}x^2 + n \cdot y^{n+1} \cdot 2x + n(n-1)y^n + y^{n+1} + n + ny^n + y^n = 0$

$= y^{n+2}x^2 + [n \cdot 2xy^{n+1} + xy^{n+1}] + [n(n-1)y^n + ny^n + y^n] = 0$

$= x^2y^{n+2} + (2x+1)xy^{n+1} + (n^2 - n + 1 + n)y^n = 0$

$= x^2y^{n+2} + (2x+1)xy^{n+1} + (n^2+1)y^n = 0$