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Assignment

1) If $y = e^{x^2+x}$
show that $y'' = y'(2x+1) + 2y$
and, hence, prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

If $y = e^{x^2+x}$

$\frac{dy}{dx} = y'$

Let $y = e^u$, $dy/dx = y'$

$= \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = y' = e^u(2x+1)$

$y' = (2x+1)e^u$

$y' = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y''$

$y' = (2x+1)e^{x^2+x}$

let $u = 2x+1$ $\qquad v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\qquad \frac{dv}{dx} = 2x+1e^{x^2+x}$

$\therefore \frac{d^2y}{dx^2} = y'' = V\frac{du}{dx} + u\frac{dv}{dx}$

$= 2(e^{x^2+x}) + 2x+1((2x+1)e^{x^2+x})$

since $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2(y) + (2x+1)(y')$

$y'' = 2y + (2x+1)y'$

$y'' = y'(2x+1) + 2y$

Hence prove that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using leibnitz theorem

Let $y'' = \text{sub}1$, $-y'(2x+1) = \text{sub}2$, $-2y = \text{sub}3$

for sub1

$u = y''$ $\quad v = 1$

$U^n = y^{n+2}$ $\quad v' = 0$

$= U^n v + nu^{n-1}v'$

$= y^{n+2}(1) + n(y^{(n+1)})(0)$

$= y^{n+2}$

for sub 2

$-y'(2x+1)$

$u = y'$ $\quad v = -(2x+1)$

$U^n = y^{n+1}$ $\quad v' = -2$

$v^n = 0$

$= U^n v + nu^{n-1}v' + n(n-1)u^{n-2}v^n$

$= -y^{n+1}(+2x+1) + n(y^n)(-2)$

$= -(2x+1)y^{n+1} - 2ny^n$

for sub3

$-2y$

$u = y$ $\quad v = -2$

$U^n = y^n$ $\quad v' = 0$

$= U^n v + nu^{n-1}v'$

$= -2y^n$

$\therefore y^{n+2} - (2x+1)y^{n+1} - 2ny^n - 2y^n = 0$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$ (Proven)

2) Using the leibnitz theorem given that

i) $y = x^3 e^{4x}$, determine $y^{(5)}$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Sol^n

i) $y = x^3 e^{4x}$

determine $y^{(5)}$

$u = e^{4x}$ $\qquad v = x^3$

$u' = 4e^{4x}$ $\qquad v^{(1)} = 3x^2$

$u^{(2)} = 16e^{4x}$ $\qquad v^{(2)} = 6x$

$u^{(3)} = 64e^{4x}$ $\qquad v^{(3)} = 6$

$u^{(4)} = 256e^{4x}$ $\qquad v^{(4)} = 0$

$u^{(5)} = 1024e^{4x}$ $\qquad v^{(5)} = 0$

$$y^5 = u^5 v + n u^4 v' + \frac{n(n-1)}{2!} u^3 v^2 + \frac{n(n-1)(n-2)}{3!} u^2 v^3 + \frac{n(n-1)(n-3)(n-3)}{4!} u' v^4$$
$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{3!} u v^5$$

$$y^5 = [1024e^{4x}(x^3)] + [5(256e^{4x})3x^2] + [\tfrac{5\times4}{2}\times 64e^{4x}\times 6x] + [\tfrac{5\times4\times3}{3\times2}\times 16e^{4x} \times 6]$$
$$+ [0][0]$$

$$y^5 = 1024e^{4x}x^3 + 1280e^{4x}(3x^2) + 640e^{4x}(6x) + 160e^{4x}(6)$$

$$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$$\underbrace{x^2y''}_{\text{sub 1}} + \underbrace{xy'}_{\text{sub 2}} + \underbrace{y}_{\text{sub 3}} = 0$$

for sub 1, x^2y''

$u = y''$ $\qquad v = x^2$

$u^n = y^{n+2}$ $\qquad v' = 2x$

$v'' = 2$

$v''' = 0$

$$y^n = u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v'' + \frac{n(n-1)(n-1)}{3!} u^{n-3}$$

$$y^n = y^{n+2}x^2 + ny^{n+1}(2x) + \frac{n(n-1)}{2!} y^n(2) + 0$$
$$y^n = y^{n+2}x^2 + 2xny^{n+1} + n(n-1)y^n$$

for sub 2; xy'

$u = y'$ $\quad v = x$

$u^n = y^{n+1}$ $\quad v' = 1$

$v'' = 0$

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!} u^{n-2} v''$$

$$y^n = y^{n+1}x + ny(1) + 0$$
$$y^n = xy^{n+1} + ny^n$$

for sub 3; y

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$$y^n = U^n v + nu^{n-1}v'$$
$$= y^n(1) + n(y^{n-1})(0)$$
$$y^n = y^{n+2}x^2 + ny^{n+1}2x + [n(n-1)]y^n + xy^{n+1} + ny^n + y^n$$
$$y^n = x^2(y^{n+2}) + 2xn(y^{n+1}) + (n^2-n)y^n + x(y^{n+1}) + ny^n + y^n$$
$$y^n = (n^2 - n + n + 1)y^n + (2xn + x)y^{n+1} \cancel{+ x^2(y^{n+2})} + x^2(y^{n+2})$$
$$\therefore 0 = x^2 y^{n+2} + (2xn + x)y^{n+1} + (n^2+1)y^n$$
$$0 = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n$$
$$\therefore x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$