1) $l_b$ $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x+1$

$2x + 1e^u \qquad u = x^2 + x$

$\frac{dy}{dx} = 2x + 1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2 + 4x + e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \qquad y' = \frac{dy}{dx} \qquad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + e^{x^2+x}$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + e^{x^2+x}$

$\underbrace{y''}_{w_1} = \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$

$w_1 \Rightarrow u = y'' \qquad v = 1$

$u^n = y^{n+2} \qquad v = 0$

$= y^{n+2} \cdot 1 + 0$

$w_2 \Rightarrow u = y' \qquad V = 2x + 1$

$u^n = y^{n+1} \qquad v' = 2$

$u^{n-1} = y^n \qquad v = 0$

$\Rightarrow y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$\Rightarrow y^{n+1}(2x+1) + 2n(y^n)$

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$w_3$

$u = 2y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

$= 2[(y^n \cdot 1) + 0]$

$= 2y^n$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

(2) Using the leibnitz theorem given that $y = x^3 e^{4x}$ determine $y^5$

$u = e^{4x} \qquad v = x^3$

$y^5 = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + u v^5$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} 6x) + 5(4^2 e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} 6x + 80 e^{4x} 6$

$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 480 e^{4x}$

(ii) $x^2 \frac{dy^2}{dx^2} + \frac{x dy}{dx} + y = 0$

$\underbrace{x^2 y''}_{w_1} + \underbrace{xy'}_{w_2} + \underbrace{y}_{w_3} = 0$

$w_1 + w_2 + w_3 = 0$

$w_1$

$u = y'' \qquad v = x^2$

$u^n = y^{n+2} \qquad v' = 2x$

$u^{n-1} = y^{n+1} \qquad v'' = 2$

$u^{n-2} = y^n \qquad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n \cdot 2}{2!} + 0$

$\Rightarrow x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

For $w_2$

$u = y'$ $\quad v = x$

$u^n = y^{n+1}$ $\quad v' = 1$

$u^{n-1} = y^n$ $\quad v'' = 0$

$\Rightarrow y^{n+1} \cdot x + n y^n + 0$

For $w_2$

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$= y^n \cdot 1$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2n x y^{n+1} + (n^2 - n) y^n + x y^{n+1} + n y^n + y^n$

$x^2 y^{n+2} + 2n x y^{n+1} + x y^{n+1} + n^2 y^n - n y^n + n y^n + y^n$

$x^2 y^{n+2} + 2n + 1 (x y^{n+1}) + (n^2 + 1) y^n$