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15/ENG04/055

ELEE 16/EEG

1) If $y = e^{x^2 + x}$ show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2^{(n+1)} y_n$

Soln

$y = e^{x^2 + x}$

let $v = x^2 + x$

$\frac{dv}{dx} = 2x+1$

$y = e^v$

$\frac{dy}{dv} = e^v$

$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx}$

$= e^v \times 2x+1$

where $v = x^2 + x$

$\frac{dy}{dx} = e^{x^2+x}(2x+1) = 'y'$

if $y' = e^{x^2+x} \cdot (2x+1) = y'$

then $y'' = 2e^{x^2} \cdot e^{x^2+x}(2) + [e^{x^2+x} \cdot (2x+1)](2x+1)$

$y'' = 2e^{x^2+x} + e^{x^2+x} \cdot (2x+1) \cdot (2x+1)$

applying lebnitz theorem to the equation, find the nth derivative.

$y^{(n+1)} = 2(n+1)y^n + y^{(n+1)}(2x+1)$

(2) using lebnitz theorem given that $y = x^3 e^{4x}$ determine.

Soln

recall ~~that~~ lebnitz theorem ~~states that~~

$y^5 = u^5v + 5u^4v' + 10u^3v^2 + 10u^2v^3 + 5u'v^4 + v^5$

Where $v = e^{4x}$ $V = x^3$

$v' = 4e^{4x}$ $V' = 3x^2$

$v^2 = 16e^{4x}$ $V^2 = 6x$

$v^3 = 64e^{4x}$ $V^3 = 6$

$v^4 = 256e^{4x}$ $V^4 = 0$

$v^5 = 1024\, e^{4x}$

Therefore

$$y^3 = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x})6 + 3(4e^{4x})(0)$$

$$= 1024x^3 \cdot e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x}$$

$$y^3 = e^4(1024x^3 + 3840x^2 + 3840x + 960)$$

(11) $4\ \frac{x^2 dy}{dx^2} + \frac{x\, dy}{dx} + y = 0$

show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^2 = 0$

Soln

The equation can be written as $x^2y'' + xy' + y = 0$

let $u = y^2$, $U^n = y^{n+2}$

let $v = x^2$, $V' = 2x$, $V'' = 2$, $V''' = 0$

$$w. = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^n \times 2 + 0$$

$W_2 = xy'$

let $v = y'$, $U^n = y^n$

let $v = x$, $V' = 1$ and $V'' = 0$

$W_2 = y^{(n+1)} \cdot x + ny^n \times 1 + 0$

$= xy^{n+1} + ny^n$

$w_3 = y^{(n)}$

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Combining

$w = w_1 + w_2 + w_3$

$w = x^2 y^{(n+2)} + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} y^{n+2} \cdot xy' + y^{(n+1)} x$

$+ n y^n + y^n = 0$

$= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^n [(n(n-1) + n+1)] = 0$

$x^2 y^{(n+2)} + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$

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