Date ______ Page ______

NAME: ALABO UGWUEGHICHUKA

MATRIC NO: 18/ENG06/009

DEPT: MECH ENG.

COURSE: ENG 381

Assignment.

(1) If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

Soln.

$y = e^{x^2+x}$ $u = e^{x^2+x}$, $v = 1$

$u^{n} = (2x+1)^{n} e^{x^2+x}$ $v^{(1)} = 0$

$u^{(n-1)} = (2x+1)^{n-1} e^{x^2+x}$

$y^{n} = u^{n}v + nu^{(n-1)}v^{1}$ $\therefore y^{n} = (2x+1)^{n} e^{x^2+x}.1 + (2x+1)^{n-1} e^{x^2+x}.0$

$y^{n} = (2x+1)e^{x^2+x}$ let $n=1$ $\therefore y^{(1)} = (2x+1)^{1} e^{x^2+x}$; $u = e^{x^2+x}$; $v = 2x+1$

$u^{(n)} = (2x+1)^{n} e^{x^2+x}$; $v^{(1)} = 2$ $\therefore u^{n-1} = (2x+1)^{n-1} e^{x^2+x}$; $v^{(2)} = 0$.

$y^{(n)} = (2x+1)^{n} e^{x^2+x}(2x+1) + n(2x+1)^{n-1} e^{x^2+x}.2$

$= (2x+1)(2x+1)^{n} e^{x^2+x} + 2n(2x+1)^{n-1} e^{x^2+x}$

let $n=1$, $y'' = (2x+1)(2x+1)^{n} e^{x^2+x} + 2n(2x+1)^{n-1} e^{x^2+x}$

$y'' = (2x+1)(2x+1)^{1} e^{x^2+x} + 2(e^{x^2+x})$

Substitute $y = e^{\cancel{x}^2+x}$ and $y' = (2x+1)^{1} e^{x^2+x}$ in y''

$y'' = (2x+1)y' + 2y$ $\therefore y'' = y'(2x+1) + 2y$

ii) $y^{(2)} = y^{(1)}(2x+1) + 2y$ $\therefore y^{2} - y^{(1)}(2x+1) - 2y = 0$

$w_1 = y^{(2)}$, $w_2 = -y^{(1)}(2x+1)$, $w_3 = -2y$.

$u = y^{(2)}$; $v = 1$ $u = -y^{(1)}$ $v = 2x+1$, $u = y'$, $v = 2$

$u^{n} = y^{(n+2)}$, $v^{(1)} = 0$, $u^{n} = y^{(n+1)}$, $v^{(1)} = 0$

$y^{(n-1)} = y^{n}$ $v^{(1)} = 0$

Ruita

Date: ___\ ___\ ___ Page: ____

$y^{(n)=0} = u^{(n)} v + n u^{(n-1)} v^{(1)}$

$y^{(n)=0} = y^{(n+2)}(1) - y^{(n+1)}(2x+1) + n(-y^{n})(2x) + 2(-y^{n})$

$y^{(n)=0} = y^{(n+2)} - (2x+1) y^{(n+1)} - 2xn(-y^{n}) - 2(-y^{n})$

$\cancel{y^{(n)} = 0 = y^{(n+2)} - (2}$ $y^{n=0} + 2n(-y^{n}) + 2(-y^{n}) + (2x+1) y^{(n+1)} = y^{(n+2)}$

$\therefore y^{(n+2)} = (2x+1) y^{n-1} + 2(n+1) y^{(n)}$

2) Using Leibnitz theorem;

i) $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$, $v = x^3$

$u^{n} = 4^{n} e^{4x}$, $v^{(1)} = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$, $v^{(2)} = 6x$

$u^{(n-2)} = 4^{(n-2)} . e^{4x}$, $v^{(3)} = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$, $v^{(4)} = 0$

$$y^{(n)} = u^{(n)} v + n u^{(n-1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^{3}$$

$$y^{(n)} = 4^{n} e^{4x} x^3 + n 4^{(n-1)} e^{4x} . 3x^2 + \frac{n(n-1)}{2!} . 4^{(n-2)} e^{4x} 6x + \dots +$$

$$\frac{n(n-1)(n-2)}{3!} . 4^{(n-3)} e^{4x} . 6$$

$$y^{(5)} = 45 e^{4x} . x^3 + 5 . 4^{(4)} e^{4x} . 3x^2 + \frac{5(4)}{2 \times 1} + 4^{(3)} e^{4x} . 6x + \frac{5(4)(3)}{3 \times 2 \times 1} \times 4^{(2)} e^{4x}$$

$$y^{(5)} = 1024 e^{4x} . x^3 + 3840 e^{4x} . x^2 + 3840 e^{4x} + 960 e^{4x}$$

$$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$y^{(5)} = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15$$

ii) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1) y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Date: \ \ Page:

also written as;

$x^2y'' + xy' + y = 0$; $x^2y^{(2)} + xy^{(1)} + y^{(0)} = 0$

$w_1 = x^2y^{(2)}$, $w_2 = xy^0$, $w_3 = y(0)$

for w_1: for w_2: for w_3:

$u = y^{(2)}$, $v = x^2$ $u = y^{(1)}$, $v = x$ $u = y^{(0)}$, $v = 1$

$u^n = y^{(n+2)}$, $u' = 2x$ $u^n = y^{(n+1)}$, $u' = 1$ $u^n = y^n$, $v^{(1)} = 0$

$u^{(n-1)} = y^{(n+1)}$, $v'' = 2$ $u^{n-1} = y^{(n)}$, $v'' = 0$

$u^{(n-2)} = y^n$, $v''' = 0$

$y^n = y^{(n+2)} \cdot x^2 + n\,y^{(n+1)} \cdot 2x + \frac{n(n-1)\,y^n \times 2}{2 \times 1} + y^{(n+1)} \cdot x + n y^n \cdot 1 + y^n$

$y^n = y^{(n+2)} \cdot x^2 + 2xn\,y^{(n+1)} + \underline{n(n-1)}\,y^{(n)} + xy^{(n+1)} + ny^n + y^n$

$y^n = x^2y^{(n+2)} + 2nxy^{(n+1)} + (n^2-n)\,y^{(n)} + xy^{(n+1)} + y^n + ny^n$

$y^n = x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2+1-n+n) = 0$

$y^n = x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2+1) = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2+1) = 0$