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15/ENG03/002
Civil ENGINEERING

ENG 382 Assignment 3

(1) $y = e^{x^2+x}$ ——— (i)

$y^n = a^n e^{ax}$

$y' = (2x+1)e^{x^2+x}$ ——— (ii)

$u = 2x+1 \qquad V = e^{x^2+x}$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = (2x+1)e^{x^2+x}$

Using Product rule

$y'' = u\frac{dv}{dx} + V\frac{du}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

from eqn (i) and (ii)

$y'' = y'(2x+1) + 2y$ ---

$y^{(2)} = y^{(1)}(2x+1) + 2y$

$W_1 = y^{(2)} \qquad W_2 = y^{(1)}(2x+1) \qquad W_3 = 2y$

$u = y^{(2)} \quad v = 1 \qquad u = y^{(1)} \quad v = 2x+1 \qquad u = y \quad v = 2$

$u^n = y^{(2+n)} \qquad u^n = y^{(1+n)} \quad v' = 2 \qquad u^n = y^n$

$u^{n-1} = y^{(n)}$

$w_1^{(n)} = w_2^{(n)} + w_3^{(n)}$

$y^n = u^{(n)}v + nu^{(n-1)}v'$

$y^{(2+n)} = y^{(1+n)} \cdot (2x+1) + n(y^{(n)}) \cdot 2 + y^{(n)}2$

$y^{(2+n)} = (2x+1)y^{(1+n)} + 2(n+1)y^{(n)}$

(2) $y = x^3 e^{4x}$ find $y^{(5)}$

$u = e^{4x} \qquad V = x^3$

$u^n = 4^n e^{4x} \qquad v' = 3x^2$

$u^{n-1} = 4^{n-1} e^{4x} \qquad v^2 = 6x$

$u^{n-2} = 4^{n-2} e^{4x} \qquad v^3 = 6$

$u^{n-3} = 4^{n-3} e^{4x}$

$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}v^2}{2!} + \frac{n(n-1)(n-2)u^{(n-3)}}{3!} + \cdots$

2 $y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x}) \cdot 3x^2 + \frac{5(4)(4^3 e^{4x}) \cdot 6x}{2!}$

$+ \frac{5(4)(3)(4^2 e^{4x}) \cdot 6}{3!}$

$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}$

$y^{(5)} = 64e^{4x}[16x^3 + 60x^2 + 60x + 15]$

(2) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$w_1 = x^2 y(2)$

$u = y(2) \quad v = x^2$

$u^{(n)} = y^{(2+n)} \quad v' = 2x$

$u^{(n-1)} = y^{(1+n)} \quad v^2 = 2$

$u^{(n-2)} = y^{(n)}$

$w_2 = xy(1)$

$u = y(1) \quad v = x$

$u^{(n)} = y(1+n) \quad v' = 1$

$u^{(n-2)} = y(n)$

$w_2 = xy(1)$

$u = y(1) \quad v = x$

$u(n) = y(1+n) \quad v' = 1$

$u^{(n-1)} = y(n)$

$w_3 = y$

$u = y \quad v = 1$

$u^n = y^{(n)}$

Using Leibnitz theorem

$y^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)u^{(n-2)}v^2}{2!}$

$w_1^{(n)} = y^{(2+n)} \cdot x^2 + n(y^{(1+n)} \cdot 2x) + \frac{n(n-1)y(n) \cdot 2}{2!}$

$w_1(n) = x^2 y^{(n+2)} + 2xny(1+n) + n(n-1)y(n)$

$w_2(n) = y(1+n) \cdot x + n y(n) \cdot 1$

$w_2(n) = x y(1+n) + n y(n)$

$w_3(n) = y(n) \cdot 1 = y(n)$

Adding together

$x^2 y(n+2) + 2xny(1+n) + n(n-1)y(n) + xy(1+n)$

$+ ny(n) + y(n) = 0$

$x^2 y(n+2) + xy(1+n)(2n+1) + y(n)(n(n-1)+n+1) = 0$

$x^2 y(n+2) + xy(1+n)(2n+1) + y(n)[n^2 - n + n + 1] = 0$

$x^2 y(n+2) + (2n+1) xy(1+n) + (n^2+1) y(n).$