Akinola Olumayowa David
15/ENG06/008
Engineering Math

①

H $y = e^{x^2} + x$

$u = x^2 + x$

$\frac{\partial u}{\partial x} = 2x + 1$

$y = e^u$

$\frac{\partial y}{\partial x} = e^u$

$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \times \frac{\partial u}{\partial x}$

$= e^u \times 2x + 1$

$2x + 1e^u \quad u = x^2 + x$

$\frac{\partial y}{\partial x} = 2x + 1e^{x^2+x}$

$\frac{\partial^2 y}{\partial x^2} = 2e^{x^2} + x + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2} + 4x^2 + 4x + 1e^{x^2+x}$

$\frac{\partial^2 y}{\partial x^2} = 2e^{x^2+x} + 4x^2 + 4x + 1 + e^{x^2+x}$

$y'' = \frac{\partial^2 y}{\partial x^2} \qquad y' = \frac{\partial y}{\partial x} \qquad y = e^{x^2} + x$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y' = (2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2} + x$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2} + x$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

w_3

$u_x = 2y \qquad r = 1$

$u^n = y^n \qquad r' = 0$

$= 2(y^n - 1) + 1$

$= 2y^n$

$w_1 = w_2 + w_3$

$y^{n+1} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2c) using the leibnitz theorem gives

$y = x^3 e^{4x}$ determine $y^{(5)}$

solution

$u = e^{4x} \qquad v = x^3$

$y = u^5 v^0 + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + u v^5$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) + 5(4^2 e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$

Scanned by CamScanner

$= 1024e^{4x}x^3 + 3840e^{4}x^2 + 3640e^{4x}x + 480e^{4x}$

(6) $x^2\frac{\partial y^2}{\partial x^2} + x\frac{\partial y}{\partial x} + y = 0$

$x^2y'' + xy' + y = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$w_1 \quad w_2 \quad w_3$

$w_1 + w_2 + w_3 = 0$

for w_1

$w = y''$ $\qquad v = v^2x^2$

$u^n = y^{n+2}$ $\qquad v' = 2$

$u^{n-1} = u^{n+1}$ $\qquad v'' = 2$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + n\frac{(n-1)}{2!}y^n \cdot x = 0$

$2x^2y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y$

for w_2

$w = y'$ $\qquad u = x$

$u^n = y^{n+1}$ $\qquad u = 1$

$u^{x-1} = y^n$ $\qquad u^n = 0$

$= y^{n+1} \cdot x + ny^n + 0$

for w_3

$u = y$ $\qquad v = 1$

$u^n = y^n$ $\qquad v' = 0$

$= y^{n-1}$

$w_1 + w_2 + w_3 = 0$

$x^2y^{n+2} - 2nxy^{n+1} + (n^{2-n})y^n + xy^{n+1} + ny^n + y^n$

$xy^{x-2} + 2nxy^{n+1} + xy^{n+1} + n^2y^{n+1} + ny^x + y^n$

$x^2y^{x+2} + 2n+1(xy^{n+1}) + (n^2+1)y^n$.