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15/ENG06/037
Mechanical Engineering
ENG 381

1. If $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x+1$

$y = e^u \qquad dy/du = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x+1$

$= e^u 2x+1$

when $u = x^2 + x$

$\frac{dy}{dx} = 2x+1\, e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + 4x^2+4x+e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \qquad y' = dy/dx \qquad y = e^{x^2+x}$

$y'' = y'(2x+1)+2y$

$y'' = 2e^{x^2+x} + 4x^2+4x+1\, e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)\, e^{x^2+x}$

$= 4x^2 + 4x + e^{x^2+1}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1 + e^{x^2+x}$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + e^{x^2+x}$

$w_1 = y''$

$w_2 = y'(2x+1)$

$w_3 = 2y$

For w_1

$u = y'' \qquad v = 1$

$u^n = y^{n+2} \qquad v = 0$

$= y^{n+2} \cdot 1 + 0$

For w_2

$u = y' \qquad v = 2x + 1$

$u^n = y^{n+1} \qquad v' = 2$

$u^{n-1} = y^n \qquad v = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

For w_3

$u = y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

$2[(y^n \cdot 1) + 0]$

$2y^n$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2. Using the Leibnitz theorem given that

$$y = x^3 e^{4x}$$

Determine $y^{(5)}$

$V = x^3$	$U = e^{4x}$
$V' = 3x^2$	$U' = 4e^{4x}$
$V'' = 6x$	$U'' = 16e^{4x}$
$V''' = 6$	$U''' = 64e^{4x}$
$V^{(4)} = 0$	$U^{(4)} = 256e^{4x}$
$V^{(5)} = 0$	$U^{(5)} = 1024e^{4x}$

$$y^{(5)} = U^5V + nU^4V' + \frac{n(n-1)}{2!}U^3V^2 + \frac{n(n-1)(n-2)}{3!}U^2V^3 + \frac{n(n-1)(n-2)(n-3)}{4!}$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{5} \cdot UV^5$$

$$y^{(5)} = [1024\, e^{4x}(x^3)] + [5(256e^{4x})3x^2] + [\tfrac{5\times4}{2} \times 64e^{4x} \times 6x] +$$
$$[\tfrac{5\times4\times3}{3\times2} \times 16e^{4x} \times 6] + 0$$

$$y^{(5)} = 1024e^{4x}\cdot x^3 + 3840e^{4x}\cdot x^2 + 3840e^{4x}\cdot x + 960e^{4x}$$
$$y^{(5)} = e^{4x}[1024x^3 + 3840x^2 + 3840x + 960]$$

ii $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$$x^2y'' + xy' + y = 0$$

$W_1 = x^2y''$ $\quad W_2 = xy'$ $\quad W_3 = y$

For W_1,

$U = y''$	$V = x^2$
$U^n = y^{n+2}$	$V' = 2x$
$U^{n-1} = y^{n+1}$	$V'' = 2$
$U^{n-2} = y^n$	$V''' = 0$

$$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n \cdot 2}{2!} + 0$$

$$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$$

For W_2

$U = y'$ $\quad V = x$

$U^n = y^{n+1}$ $\quad V' = 1$

$U^{n-1} = y^n$ $\quad V'' = 0$

$$= y^{n+1} \cdot x + ny^n + 0$$

For W_3

$U = y$ $\quad V = 1$

$U^n = y^n$ $\quad V' = 0$

$$= y^n \cdot 1$$

$W_1 + W_2 + W_3 = 0$

$$x^2 y^{n+2} + 2nx y^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n$$

$$x^2 y^{n+2} + 2nx y^{n+1} + xy^{n+1} + n^2 y^n - ny^n + ny^n + y^n$$

$$x^2 y^{n+2} + 2n+1\,[xy^{(n+1)}] + [n^2+1]y^n$$