$w_1 + w_2 + w_3 = 0$

$y^n = u^n \cdot v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2$

substituting the values obtained,

$= y^{(n+2)} x^2 + n\, y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2!} \cdot y^{(n)} \cdot 2 + y^{(n+1)} \cdot x + n \cdot y^{(n)} \cdot 1 + y^{(n)} \times 1 = 0$

$= x^2 y^{n+2} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$

$= x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)} = 0$

$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n(n-1)+n+1)y^{(n)} = 0$

$= x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2 - n + n + 1) y^{(n)}$

$= x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

ENH 3x1 (Assignment 3)

$y = e^{x^2+x}$

$y' = 2x+1(e^{x^2+x})$

$y'' = e^{x^2+x}(2)+(x+1)(2x+1)e^{x^2+x}$

$y'' = 2y+(2x+1)y'$

$\underbrace{y''}_{w_1} = \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$

$w_1 = y^{(2)}$

$u = y''$ $v = 1$

$u^n = y^{(n+2)}$

$w_2 = y'(2x+1)$

$u = y'$ $v = (2x+1)$

$u^n = y^{(n+1)}$ $v' = 2$

$u^{(n-1)} = y^{(n)}$

$w_3 = 2y$

$u = y$ $v = 2$

$u^n = y^{(n)}$

$w_1 = w_2 + w_3\,[y^n = u^{(n)}v + nu^{(n-1)}v']$

$y^{(n+2)} = (2x+1)y^{(n+1)} + ny^{(n)}\cdot 2 + y^{(n)}\,2$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^{(n)} + 2y^{(n)}$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2. $y = x^3e^{4x}$ $v = x^3$

$u = e^{4x}$ $v' = 3x^2$

$u^n = 4^ne^{4x}$ $v'' = 6x$

$u^{(n-1)} = 4^{(n-1)}e^{4x}$ $v''' = 6$

$u^{(n-2)} = 4^{(n-2)}e^{4x}$ $v^{(4)} = 0$

$u^{(n-3)} = 4^{(n-3)}e^{4x}$

$y^n = u^nv + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \frac{n(n-2)(n-1)}{3!}u^{n-3}v^3$

$y^5 = 4^5e^{4x}x^3 + 5(4^{(5-1)}e^{4x})\,3x^2 + \frac{5(5-1)}{2!}(4^{(5-2)}e^{4x})\cdot 6x + \frac{5(5-1)(5-2)}{3!}(4^{(5-3)}e^{4x})\cdot 6$

$y^5 = 1024e^{4x}x^3 + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$

~~$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$~~

$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$

$y^5 = 64e^{4x}(16x^3+60x^2+60x+15)$

(1) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$\underbrace{x^2y^{(2)}}_{w_1} + \underbrace{xy^{(1)}}_{w_2} + \underbrace{y}_{w_3} = 0$

$w_1 = x^2y^{(2)}$

$u = y^{(2)}$ $v = x^2$

$u^n = y^{(n+2)}$ $v' = 2x$

$u^{n-1} = y^{(n+1)}$ $v'' = 2$

$u^{(n-2)} = y^{(n)}$ $v^{(3)} = 0$

$w_2 = xy^{(1)}$

$u^n = y^{(n+1)}$ $v = x$

$u^{(n-1)} = y^{(n)}$ $v' = 1$

$v^{(2)} = 0$

$w_3 = y$ $v = 1$

$u = y$

$u^n = y^n$

$w_1 + w_2 + w_3\,[y^n = u^nv + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v''] \; y^{(n+2)}x^2 + ny^{(n+1)}2x +$

$\frac{n(n-1)}{2!}y^{(n)}2 + y^{(n+1)}x + ny^{(n)}1 + y^{(n)}$

$y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^{(n)}2 +$