ASSIGNMENT 3

1. $y = e^{x^2+x}$ ∴ Show that $y'' = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$

$y'' = \frac{d^2y}{dx^2}$ $\quad y' = \frac{dy}{dx}$

∴ $\frac{dy}{dx} = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$\frac{d^2y}{dx^2} = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$\frac{d^2y}{dx^2} = (4x^2+4x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = (4x^2+4x+3)e^{x^2+x}$

~~C4x²~~

$(4x^2+4x+3)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x^2+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+3)e^{x^2+x}$

$= (4x^2+4x+4)e^{x^2+x}$

$(4x^2+4x+3)e^{x^2+x} = (4x^2+4x+3)e^{x^2+x}$

∴ $y'' = y'(2x+1) + 2y$

i. $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

let $w = y''$ $\quad v = 1$ $\quad v' = 0$ $\quad u = y''$ $\quad u^n = y^{n+2}$

$w^n = u^n v + n u^{n-1} v'$

$= y^{(n+2)} + 0$

let $w = -y'(2x+1)$

$v = 2x+1$ $\quad v' = 2$ $\quad v'' = 0$ $\quad u = -y'$ $\quad u^n = -y^{n+1}$

$w^n = u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v''$

$= -y^{n+1}(2x+1) + n(-y^{n+1-1})(2) + 0$

$= -y^{n+1}(2x+1) + 2n(-y^n)$

let $w = -2y$

$v = -2 \quad v' = 0$

$u = y \quad u^n = y^n$

$w^n = u^n v + n u^{n-1} v'$

$= y^n - 2 + 0$

$= -2y^n$

$\therefore y^n = y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^2$

$y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n-1)$

2. $y = x^3 e^{4x}$ Find y^5

$v = x^3 \quad v' = 3x^2, \ v'' = 6x, \ v''' = 6, \ v^4 = 0$

$u = e^{4x}, u' = 4e^{4x}, u^2 = 16e^{4x}, u^3 = 64e^{4x} \quad u^4 = 256e^{4x} \ u^5 = 1024e^{4x}$

$y^n = u^n v + n v^{-n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 +$

$\frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^4$

$y^5 = 1024e^{4x}(x^3) + 15x^2(256e^{4x}) + 60x(64e^{4x}) + 60(16e^{4x})$

$y^5 = x^3 1024e^{4x} + x^2 3840e^{4x} + x3840e^{4x} + 960e^{4x}$

$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

2i $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + xy + y = 0$

let $w = x^2 y''$

$v = x^2, \ v' = 2x, \ v'' = 2, \ v''' = 0$

$u = y'' \quad u^n = y^{n+2}$

$w^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-2} v'''$

$= y^{n+2}(x^2) + n(y^{n+2})(2x) + \frac{n(n-1)}{2!}(y^{n+2-2})(2)$

$= x^2 y^{n+2} + 2xn(y^{n+1}) + n(n-1)(y^n)$

let $w = xy$

$v = x \quad , \quad v' = 1 \quad , \quad v'' = 0$

$u = y' \qquad u^n = y^{n+1}$

$w^n = y^{n+1}(x) + n(y^{n+1-1})(1) + 0$

$\quad = -xy^{n+1} + ny^n$

let $w = y$

$v = 1 \qquad v' = 0$

$u = y \qquad u^n = y^n$

$w^n = y^n$

$y^n = x^2 y^{(n+2)} + 2xn(y^{n+1}) + n(n-1)(y^n) + xy^{n+1} + ny^n + y'$

$x^2 y(n+2) + (2n+1)\dot{x}y^{(n+1)} + (n^2 - n + n + 1)y^n = 0$

$x^2 y(^{n-2}) + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$