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15/ENG07/016

PETROLEUM ENGINEERING

Question 07

$y = e^{x^2+x}$

Show that:

$y'' = y'(2x+1) + 2y$.

Prove that:

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution:

$y = e^{x^2+x} \longrightarrow$ equ. (i)

$y' = (2x+1)e^{x^2+x} \longrightarrow$ equ. (ii)

Obtaining y'' by product rule

Let $u = (2x+1)$, $v = e^{x^2+x}$

$\frac{du}{dx} = 2$, $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

but $\frac{dy'}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= e^{x^2+x} \cdot 2 + (2x+1)(2x+1)e^{x^2+x}$

$\therefore y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

but $y' = (2x+1)e^{x^2+x}$, $y = e^{x^2+x}$

$\therefore \cancel{y'' = (2x+1)} \; (2x+1)y'$

$\therefore y'' = y'(2x+1) + 2y$.

Considering $y'' = y'(2x+1) + 2y$

Let $a_1 = y''$

$a_1' = y'''$

$a_1'' = y^{iv}$

$\therefore a_1^n = y^{(n+2)}$

Let $a_2 = y'(2x+1)$

$u = y'$, $v = 2x+1$

$u' = y''$, $v' = 2$

$u'' = y'''$, $v'' = 0$

$\therefore u^n = y^{n+1}$

but $a_2^n = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v''$

$= y^{n+1} \cdot (2x+1) + n \cdot y^n \cdot 2 + \cancel{\frac{n(n-1)}{2} y^{n-1} \cdot 0} \rightarrow 0$

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$\therefore a_2^n = (2x+1)y^{(n+1)} + 2ny^n$

Let $a_3^n = 2y$

$a_3' = 2 \qquad u = y \qquad v = 2$

$a_3'' = 0 \qquad u^n = y^n \qquad v' = 0$

$a_3^n = u^n v^0 + n u^{n-1} v'$

$\therefore a_3^n = y^n \cdot 2 + 0$

Hence, $a_3^n = 2y^n$

but $a_1^n = a_2^n + a_3^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^n + 2y^n$

Re-arranging and collecting like terms

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$.

Question 2

Using the Leibnitz theorem, given that

i $y = x^3 e^{4x}$, determine y^5.

ii $x^2 y'' + xy' + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Solution:

i $y = x^3 e^{4x}$ determine y^5

Let $u = e^{4x}$, $v = x^3$

$u' = 4e^{4x} \qquad v' = 3x^2$

$u'' = 16e^{4x} \qquad v'' = 6x$

$u''' = 64e^{4x} \qquad v''' = 6$

$u^{iv} = 256e^{4x} \qquad v^{iv} = 0$

$\therefore u^n = 4^n e^{4x}$

but $y^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v''' + \frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^{iv}$

$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{5-1} e^{4x} 3x^2 + \frac{5 \cdot \cancel{4}}{\cancel{2}} \cdot 4^{5-2} e^{4x} \cdot 6x + \frac{5 \cdot 4 \cdot 3}{\cancel{6}} 4^{5-3} e^{4x} \cdot 6 + 0$

$\therefore y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x}$.

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ii $x^2y'' + xy' + y = 0$

Let $a_1 = x^2y''$

$u = y''$ $\quad v = x^2$

$u' = y'''$ $\quad v' = 2x$

$u'' = y^{iv}$ $\quad v'' = 2$

$u''' = y^{v}$ $\quad v''' = 0$

$\therefore u^n = y^{(n+2)}$

but $a_1^n = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v'' + \frac{n(n-1)(n-2)}{3!}u^{n-3}v'''$

$a_1^n = y^{(n+2)} \cdot x^2 + n \cdot y^{n+1} \cdot 2x + \frac{n(n-1)}{2} \cdot y^n \cdot 2 + 0$

$\therefore a_1^n = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$

Let $a_2 = xy'$

$u = y'$ $\quad v = x$

$u' = y''$ $\quad v' = 1$

$u'' = y'''$ $\quad v'' = 0$

$\therefore u^n = y^{(n+1)}$

but $a_2^n = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v''$

$a_2^n = y^{(n+1)} \cdot x + n \cdot y^n \cdot 1 + 0$

$\therefore a_2^n = xy^{(n+1)} + ny^n$

Let $a_3 = y$

$a_3^n = y^n$

but $a_1^n + a_2^n + a_3^n = 0$

$x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n = 0$

Re-arranging and collecting like terms:

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$.

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