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DEPARTMENT: ELECTRICAL/ELECTRONICS ENGINEERING

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ASSIGNMENT 3

1) $y = e^{x^2+x}$

Using chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$u = x^2 + x$$

$$\frac{du}{dx} = 2x + 1$$

$$y = e^u$$

$$\frac{dy}{du} = e^u$$

$$\frac{dy}{dx} = e^u \times (2x+1) = (2x+1)e^u$$

$$\frac{dy}{dx} = (2x+1)e^{x^2+x} \Rightarrow y' = (2x+1)e^{x^2+x}$$

Since $y = e^{x^2+x}$ and

$$\frac{dy}{dx} = (2x+1)e^{x^2+x}$$

To find $y'' \Rightarrow \frac{d^2y}{dx^2}$, we use product rule
 $\Rightarrow u \frac{dv}{dx} + v \frac{du}{dx}$

$$u = e^{x^2+x}$$

$$v = 2x+1$$

$$\frac{du}{dx} = (2x+1)e^{x^2+x}$$

$$\frac{dv}{dx} = 2$$

$$\frac{d^2y}{dx^2} = e^{x^2+x}(2) + (2x+1)(2x+1)e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$$

Recall: $y = e^{x^2+x}$ and $y' = (2x+1)e^{x^2+x}$

$$\therefore y'' = y'(2x+1) + 2y$$

$$y'' = \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$$

w₁:

$$u = y^{(2)}$$

$$u^{(n)} \triangleq y^{(n+2)}$$

w₂:

$$u = y^{(1)} \quad v = 2x+1$$

$$u^{(n)} \triangleq y^{(n+1)} \quad v^{(n)} = 2$$

w₃:

$$u = y \quad v = 2$$

$$u^{(n)} \triangleq y^n \quad v^{(n)} = 0$$

$$W_1 = W_2 + W_3$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^n 2 + y^n 2$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + \cancel{2ny^n} + 2y^n$$

2i) $y = x^3 e^{4x}$

$$u = e^{4x}$$

$$u^{(n)} = 4^n e^{4x}$$

$$u^{(n-1)} = 4^{n-1} e^{4x}$$

$$u^{(n-2)} = 4^{n-2} e^{4x}$$

$$u^{(n-3)} = 4^{n-3} e^{4x}$$

$$u^{(n-4)} = 4^{n-4} e^{4x}$$

$$v = x^3$$

$$v^{(0)} = x^3$$

$$v^{(1)} = 3x^2$$

$$v^{(2)} = 6x$$

$$v^{(3)} = 6$$

$$v^{(4)} = 0$$

$$y^{(n)} = u^{(n)} v^{(0)} + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^{(3)}$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{(4)}$$

$$y^{(n)} = 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} \cdot 6x$$

$$+ \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} e^{4x} \cdot 0$$

$$y^{(n)} = 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^{(n)} = 8^{4x} [4^n x^3 + n 4^{n-1} 3x^2 + n(n-1) 4^{n-2} 3x + n(n-1)(n-2) 4^{n-3}]$$

when $n=5$

$$y^{(5)} = 8^{4x} [(4^5)(x^3) + (5)(4^4)(3x^2) + (5)(4)(4^3)(3x) + (5)(4)(3)(4^2)]$$

$$y^{(5)} = 8^{4x} [1024x^3 + 3840x^2 + 3840x + 960]$$

$$\text{ii) } x^2 \frac{d^2y}{dx^2} + xy' + y = 0$$

$$\Rightarrow x^2 y'' + xy' + y = 0$$

$$\Rightarrow x^2 y^{(2)} + xy^{(1)} + y = 0$$

$$w_1 = x^2 y^{(2)}$$

$$u = y^{(2)}$$

$$v = x^2$$

$$u^{(n)} = y^{(n+2)}$$

$$v^{(1)} = 2x$$

$$u^{(n-1)} = y^{(n+1)}$$

$$v^{(2)} = 2$$

$$u^{(n-2)} = y^{(n)}$$

$$v^{(3)} = 0$$

$$w_1^{(n)} = y^{(n+2)} x^2 + ny^{(n+1)} 2x + \frac{n(n-1)}{2!} y^{(n)} 2$$

$$w_2^{(n)} = y^{(n+1)} x + ny^{(n)}$$

$$w_3^{(n)} = y^{(n)}$$

$$\Rightarrow x^2 y^{(n+2)} + 2xy^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$$

$$\Rightarrow x^2 y^{(n+2)} + 2xy^{(n+1)} + xy^{(n+1)} + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)} = 0$$

$$\Rightarrow x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n(n-1) + n+1) = 0$$

$$\Rightarrow x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2 - n + n+1) = 0$$

$$\Rightarrow x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 + 1)y^{(n)} = 0$$

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