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15/ENG03/013
CIVIL ENG

Question 1

If $y = e^{x^2+x}$
show that
$y'' = y'(2x+1) + 2y$

Soln

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2 \cdot e^{x^2+x}$

recall that

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

then

$y'' = (2x+1) \cdot y' + 2 \cdot y$

$\therefore y'' = y'(2x+1) + 2y$ ------ C

Then Proving that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

Bringing Leibnitz theorem to solve (a)

$-y'' + y'(2x+1) + 2y = 0$

$\downarrow \quad \downarrow \quad |$

$w_1 \quad w_2 \quad w_3$

$w_1 = -y'' = \frac{d^2y}{dx^2}$

then ;

$v = y''$ $\quad r = 1$

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$v' = y^{(n+2)}$ $\quad u' = 0$

$w_2 = y'(2x+1)$

$v = y'$ $\quad u = 2x+1$

$v' = y^{(n+2)}$ $\quad u' = 2$

$u'' = 0$

$w_3 = 2y$

$v = y$ $\quad u = 2$

$v' = y'$ $\quad u' = 0$

Combining all 3 terms

Therefore

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^{(n)} + 2y^{(n)}$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$ //

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Q2

Using Leibnitz theorem

if $y = x^3 e^{4x}$

determine $y^{(5)}$

soln

take $u = e^{4x}$

$v = x^3$

$$y^n = u^n v + n u^{(n-1)} v^1 + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 \ldots$$

$u^n = 4^n e^{4x}$ $\quad v = x^3$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad v' = 3x^2$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad v'' = 6x$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad v''' = 6$

$v^{iv} = 0$

$$y^{(n)} = 4^n e^{4x} x^3 + n4^{(n-1)} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x} 6x + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6$$

$$y^{(n)} = x^3 4^n e^{4x} + 3x^2 n 4^{(n-1)} e^{4x} + 3x n(n-1) 4^{(n-2)} + n(n-1)(n-2) 4^{(n-3)} e^{4x}$$

Putting $n = 5$

$$y^{(5)} = x^3 4^{(5)} e^{4x} + 3x^2 (5) 4^{(5-1)} e^{4x} + 3x(5)(5-1) 4^{(5-2)} + 5(5-1)(5-2) 4^{(5-3)} e^{4x}$$

$$= 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$y^{(5)} = e^{4x} [1024x^3 + 3840x^2 + 3840x + 960]$$

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iii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Soln

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$\underbrace{x^2y''}_{w_1} + \underbrace{xy'}_{w_2} + \underbrace{y}_{w_3} = 0 \Rightarrow x^2y^2 + xy' + y = 0$

$w_1 : x^2y''$

$u = y^2 \qquad v = x^2$

$u^n = y^{(n+2)} \qquad v' = 2x$

$u^{(n-1)} = y^{(n+1)} \qquad v'' = 2$

$u^{(n-2)} = y^n \qquad v''' = 0$

$w_1 = y(n+2)\cdot x^2 + n\,y^{(n+1)}\cdot 2x + \frac{n(n-1)}{2!}y^n\cdot \cancel{2}$

$= x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$.

$w_2 : xy'$

$u = y' \qquad v = x$

$u^n = y^{(n+1)} \qquad v' = 1$

$u^{(n-1)} = y^n \qquad v'' = 0$

$w_2 = xy^{(n+1)} + ny^{(n)}$

$w_3 : y$

$u^n = y^{(n)}$

$= x^2y^{(n+2)} + 2xny^{(n+1)} + n^{(n-1)}y^{(n)} + xy^{(n+1)} + ny^n + y^n = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n[n(n-1) + n + 1] = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$