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ENG 381 ASS III

1)

If $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x+1$

$2x+1e^u \qquad u = x^2+x$

$\frac{dy}{dx} = 2x+1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2+4x+1 + e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \qquad y' = \frac{dy}{dx} \qquad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2+4x+1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1)+2y = 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$$y'' = y'(2x+1) + 2y$$

$\downarrow$ w_1 $\quad$ w_2 $\quad$ $\downarrow$ w_3

w_1

$u = y''$ $\qquad v = 1$

$u^n = y^{n+2}$ $\qquad v = 0$

$= y^{n+2} \cdot 1 + 0$

w_2

$u = y'$ $\qquad v = 2x+1$

$u^n = y^{n+1}$ $\qquad v' = 2$

$u^{n-1} = y^n$ $\qquad v = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

w_3

$u = \cancel{x} y$ $\qquad v = 1$

$u^n = y^n$ $\qquad v' = 0$

$= 2[y^n - 1) + 0]$

$= 2y^n$

$w_1 = w_2 + w_2$

$y^{n+1} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a) Using the leibnitz theorem gives that $y = x^3 e^{4x}$ determine $y^{(5)}$

Solution

$u = e^{4x}$ $\qquad v = x^3$

$y^{(5)} = u^{(5)}v + 5u^4v' + 10u^3v'' + 10u^2v^3 + 5u'v^4 + uv^5$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) +$
$5(4^2 e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$
$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 480 e^{4x}$

(iv) $x^2 \frac{dy^2}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$w_1 \quad w_2 \quad w_3$

$w_1 + w_2 + w_3 = 0$

for w_1

$u = y''$ $\quad v = \cancel{x^2}\, x^2$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)}{2!} y^n \cdot x = 0$

$2x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

for w_2

$u = y'$ $\quad u = x$

$u^n = y^{n+1}$ $\quad u' = 1$

$u^{n-1} = y^n$ $\quad u'' = 0$

$= y^{n+1} \cdot x + ny^n + 0$

for w_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$= y^n \cdot 1$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n$

$xy^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2 y^n - \cancel{ny^n} + \cancel{ny^n} + y^n$

$x^2 y^{n+2} + 2n+1(xy^{n+1}) + (n^2+1)y^n$