Name: Akinyemi Opeyemi Emmanuel
Matric No: 151611001002
Department: Chemical Engineering

① $y = e^{x^2+x}$ ② $y = x^3 e^{4x}$ ③ $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Solution 1. I

① $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

Leibniz

$y^n = y'(2x+1) + 2y$

$y^{(n+2)} = y^{(n+1)}(2x+1)(n+1)y^{n-2}$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

② $y = x^3 e^{4x}$

$y^n = u^n v + n u^{n-1}v^1 + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)v^3}{3!}$

$+ \ldots$ Let $x^3 = v$ and $u = e^{4x}$

$y^5 = (4)^5 e^{4x} \cdot x^3 + 5(4)^4 e^{4x} \cdot 3x^2 + 10(4)^3 e^{4x} \cdot 6x$

$+ 10(4)^2 e^{4x} \cdot 6 + 0$

$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} +$

$960e^{4x} + 0$

$y^5 =$ [illegible]

$y^5 = e^{4x}[1024x^3 + 3840x^2 + 3840x + 960]$

3. $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$x^2y^{(2)} + xy^{(1)} + y = 0$

Using Leibnitz theorem

$y^n = y^{(n+2)} \cdot x^2 + n \cdot 2x\, y^{(n+1)} + \frac{2n(n-1)y^n}{2!}$

$+ y^{(n+1)} \cdot x + ny^{n-1} + y^n$

$y^n = x^2y^{(n+2)} + 2nxy^{(n+1)} + n(n-1)y^n + xy^{(n+1)}$

$+ ny^{(n)} + y^{(n)}$

$y^n = y^{(n+2)}(x^2) + y^{(n+1)}(2xn + x) + y^{(n)}$

$(n(n-1) + n + 1)$

$y^n = x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2-1)y^n$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-1)y^n = 0$