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15/ENG01/006
CHEM ENGR
ENG 381

1) Using the leibnitz theorem, given that
(i) $y = x^3 e^{4x}$, determine $y^{(5)}$

sol

From leibnitz theorem

$$y^{(5)} = u^{(5)}v + 5u^{(4)}v^{(1)} + 10u^{(3)}v^{(2)} + 10u^{(2)}v^{(3)} + 5u^{(1)}v^{(4)} + uv^{(5)}$$

let $u = e^{4x}$ $\quad$ $V = x^3$

$u^{(1)} = 4e^{4x}$ $\quad$ $V^{(1)} = 3x^2$

$u^{(2)} = 16e^{4x}$ $\quad$ $V^{(2)} = 6x$

$u^{(2)} = 64e^{4x}$ $\quad$ $V^{(3)} = 6$

$u^{(4)} = 256e^{4x}$ $\quad$ $V^{(4)} = 0$

$u^{(5)} = 1024e^{4x}$ $\quad$ $V^{(5)} = 0$

$$y^{(5)} = 1024e^{4x}\cdot x^3 + 5(256e^{4x}\cdot 3x^2) + 10(64e^{4x}\cdot 6x) + 10(16e^{4x}\cdot 6) + 5\cancel{(4e^{4x}\cdot 0)}^0 + \cancel{(e^{4x}\cdot 0)}^0$$

$$y^{(5)} = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}\cdot 6x + 960e^{4x}$$

$$y^{(5)} = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)_{//}$$

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Show

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$

Using Leibnitz theorem

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}v^{(2)} + \cdots$$

let $w_1 = x^2 y^{(2)}$

let $u = y^{(2)}$ $\quad\quad$ $v = x^2$

$u^n = y^{(n+2)}$ $\quad\quad$ $v^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad\quad$ $v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\quad\quad$ $v^{(3)} = 0$

$$w_1^{(n)} = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{\cancel{2!}}y^{(n)}2 + 0 + \cdots$$

let $w_2 = xy'$

$u = y'$ $\quad\quad$ $v = x$

$u^n = y^{n+1}$ $\quad\quad$ $v^{(1)} = 1$

$u^{(n-1)} = y^n$ $\quad\quad$ $v^{(2)} = 0$

$$w_2^{(n)} = u^n v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v^{(2)} + \cdots$$

$= y^{(n+1)}x + ny^{(n)} + 0$

$= y^{(n+1)}x + ny^{(n)}$

let $w_3 = y$

$u = y$ $\quad\quad$ $v = 1$

$u^n = y^n$ $\quad\quad$ $v^{(1)} = 0$

$w_3^{(n)} = y^n + 0$

$$y^{(n)} = w_1^{(n)} + w_2^{(n)} + w_3^{(n)}$$

$= x^2 y^{(n+2)} + 2nx\, y^{(n+1)} + n(n-1) y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^n$

$= x^2 y^{(n+2)} + y^{(n+1)}(2nx + x) + y^{(n)}(n^2 - n + n + 1)$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2+1)$

If $y = e^{x^2+x}$

Show that

$y'' = y(2x+1) + 2y$

hence, prove that

$y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^n$

sol

$y = e^{x^2+x}$

$\frac{dy}{dx} = (2x+1) e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1) e^{x^2+x}$

$= e^{x^2+x} ((2x+1)(2x+1) + 2)$

$y'(2x+1) + y$

$(2x+1) e^{x^2+x} (2x+1) + 2y$

$(2x+1) e^{x^2+x} (2x+1) + 2e^{x^2+x}$

$= e^{x^2+x} ((2x+1)(2x+1) + 2)$

$y'' = y'(2x+1) + 2y$

$w_1 \qquad w_2 \qquad w_3$

w_1

$u = y^{(2)}$

$u^n = y^{n+2}$

w_2

$u = y' \qquad v = 2x+1$

$u^n = y^{n+1} \qquad v^{(1)} = 2$

$u^{n-1} = y^n \qquad v^{(2)} = 0$

$u = y$ $\quad v = x$

$u^n = y^n$ $\quad v = 0$

$w_1 = w_2 + w_3$

$y^{(n+2)} = n(y^{n+1})x + ny^{(n)} \cdot x$

$y^{(n+2)} = y^{n+1}(2x+1) + ny^n \cdot 2 + y^n 2$

$= y^{(n+1)}(2x+1) + 2ny^n + 2y^n$

$= (2x+1)y^{(n+1)} + 2(n+1)y^n$

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