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15/ENG06/059

Mechanical Engineering.

ENG 381

1 If $y = e^{x^2+x}$ show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

solution.

$y = e^{x^2+x}$

$u = e^{x^2+2}$

$v = 1$

$u^n = (2x+1)^{n-1} e^{x^2+x}$

$u^{(n-1)} = (2x+1)^{n-1} e^{x^2+x}$

$y^n = u^n v + n u^{(n-1)}$

$y^n = (2x+1)^n e^{x^2+x} + (2x+1)^{n-1} e^{x^2+x} \cdot 0$

$y^n = (2x+1)^n e^{x^2+x}$, let $n = 1$

$y^{(1)} = (2x+1) e^{x^2+x}$

$u = e^{x^2+x}$

$u^{(n)} = (2x+1)^n e^{x^2+x}$

$u^{n-1} = (2x+1)^{n-1} e^{x^2+x}$

$y^{(n)} = (2x+1)^n e^{x^2+x} + 2n(2x+1)^{n-1} e^{x^2+x}$

let $n = 1$, $y'' = (2x+1)(2x+1)^n e^{x^2+x} + 2n(2x+1)^{n-1} e^{x^2+x}$

$y'' = (2x+1)(2x+1) e^{x^2+x} + 2(e^{x^2+x})$

Sub $y = e^{x^2+x}$ and $y' = (2x+1)e^{x^2+x}$ in y''

$y'' = (2x+1)y' + 2y$

$y'' = y'(2x+1) + 2y$

$y^{(2)} = y^{(1)}(2x+1) + 2y.$

$y^{(2)} - y(2x+1) + 2y = 0$

$w_1 = -y^{(1)}(2x+1)$

$w_2 = -y^{(1)}(2x+1)$

$w_3 = 2y$

$u = y^2$

$u^n = y^{(n+2)}$ $v^{(1)} = 0$ $y^n = y$ $v^{(1)} = 2$ $\therefore$ $u^n = y^n$ $v' = 0$

$u^{n-1} = y^n$ $v'' = 0$

$y^{(n)} = 0$ $u^{(n)} + n u^{(n-1)} v^{(1)}$

$y^{(n)} = 0 = y^{(n+2)}(1) - y^{(n+1)}(2x+1) + n(-y^n)(2x) + 2(-y^n)$

x 1 1
x^2 1 2 1
x^3 1 3 3 1
x^4 1 4 6 4 1
x^5 1 5 10 10 5 1

$y^{(n)}=0 = y^{(n+2)} - (2x+1)y^{(n+1)} - 2xn(-y^n) - 2(y^n)$
$y^{(n)}=0 + 2n(y^n) + 2(y^n) + (2x+)y^{(n+1)} = y^{(n+2)}$
$y^{(n+2)} = (2x+)y^{n-1} + 2(n+1)y^{(n)}$

2) $V = x^3 e^{4x}$
Find y^5
Solution
$x = e^{4x}$ $V = x^3$
$y^5 = u^5 V + 5u^4 V^1 + 10x^3 V^{(2)} + 10u^2 V^{(3)} + 5u^1 V^{(4)} + uV^5$
$y^n = U^{(n)}V + nU^{(n-1)}V^1 + \frac{n(n-1)}{2!}U^{(n-2)}V^2 + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V^3 + \ldots$

differentiating u & V

$u = e^{4x}$	$V = x^3$
$u' = 4e^{4x}$	$V' = 3x^2$
$u'' = 16e^{4x}$	$V'' = 6x$
$u^{(3)} = 64e^{4x}$	$V^3 = 6$
$u^{(4)} = 256e^{4x}$	$V^4 = 0$
$u^{(5)} = 1024e^{4x}$	$V^5 = 0$

$y^5 = 1024e^{4x} \cdot x^3 + 5(256e^{4x})3x^2 + \frac{5(5-1)}{2!}64e^{4x} + \frac{5(5-1)(5-2)}{3!}u^{(5-3)} + \frac{5(5-1)(5-2)(5-3)}{4!}u^{(5-4)}V^4$

$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + \frac{5(4)}{2!}64e^{4x} + \frac{5(4)(3)}{3!}96e^{4x} + \frac{5(4)(3)(2)}{4!}4e^{4x}(0)$

$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x} + 960e^{4x} + 0$

(ii) $x^2 \, d^2y/dx^2 + x \frac{dy}{dx} + y = 0$

Soln

$\underbrace{x^2 y''}_{\text{sub1}} + \underbrace{x y'}_{\text{sub2}} + \underbrace{y}_{\text{sub3}} = 0$

$V =$

For sub 1

$x^2 y''$ $\quad V' = 2x$

$u = y''$ $\quad V'' = 2$

$V = x^2$ $\quad V''' = 0$

$r = 0$ to 3

there fore :-

$= U^n V^0 + n U^{n-1} V' + U^{n-2} V'' \left(\frac{n(n-1)(n-2)}{2!} \right)$

$+ U^{n-3} V''' \left(\frac{n(n-1)(n-2)(n-3)}{3!} \right)$

$= y^{(n+2)} \cdot x^2 + n \, y^{n+1} \cdot 2x + y^n \cdot 2 \left(\frac{n(n-1)}{2!} \right)$

$+ y^{(n-1)} (0) \left(\frac{n(n-1)(n-2)}{3!} \right)$

$= x^2 y^{(n+2)} + n \, 2x \, y^{n+1}) + y^n (n(n-1)(\quad))$

for sub 2

$u = y'$ $\quad V' = 1$

$V = x$ $\quad V'' = 0$

$r = 0$ to 1

:-

$= U^n V^0 + n U^{n-1} V'$ $\quad = y^{n+1} \cdot x + n \, (y^n) \cdot 1$

$= xy^{n+1} + ny^n$

for sub 3

$u = y$ $\quad V' = 0$

$V = 1$

$r = 0$

there fore

$= 1 \cdot y^n = y^n$.

Sub1 + sub2 + sub3

$= x^2 y^{(n+2)} + n\,2x\,y^{(n+1)} + y^n (n(n-1)) + x y^{(n+1)}$

$+ n y^n + y^n$

$= x^2 y^{(n+2)} + n\,2x\,y^{(n+1)} + y^n (n(n-1) + (n+1))$

$+ x y^{(n+1)}$

$= x^2 y^{(n+2)} + \cancel{n\,2x}\,x\,y^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1)$

$\cancel{= x^2 y^{(n+2)} + x y^{(n+1)}(2n+1) + y^n(2n^2+n-1)}$

$= x^2 y^{(n+2)} + x y^{(n+1)}(2n+1) + y^n(n^2+1)$