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15/ENG06/004
Mechanical Eng.

Soln $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^{n}$

$y = e^{x^2+x}$

let $u = x^2+x$

$\partial u/\partial x = 2x+1$

$y = e^{u}$

$\partial y/\partial u = e^{u}$

$\partial y/\partial x = \partial y/\partial u \times \partial u/\partial x$

$\partial y/\partial x = e^{u} \times 2x+1$

value $u = x^2+x$

$\partial y/\partial x = e^{x^2+x}(2x+1) = y'$

$y' = e^{x^2+x}(2x+1)$

$y^{n} = 2e^{x^2}e^{x^2+x}(2) + [e^{x^2+x}(2x+1)](2x+1)$

$y'' = 2e^{x^2} \cdot e^{x^2+x}(2) + [e^{x^2+x}(2x+1)](2x+1)$

Where $y = e^{x^2+x}$

$y' = e^{x^2+x}(2x+1)$

$y'' = 2(y) + y'(2x+1)$

Applying Leibnitz theorem

$y^{(n+2)} = 2(n+1)y^{n} + y(n+1)(2x+1)$

2) Using the Leibnitz theorem given that $y = x^3e^{+x}$ determine $y^{(5)}$

Soln

Recall Leibnitz theorem states that

$y^5 = u^5v + 5u^4v' + 10u^3v^2 + 10u^2v^3 + 5u'v^4 + v^5$

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$u = e^{4x}$ $\qquad$ $v = x^3$

$u' = 4e^{4x}$ $\qquad$ $v' = 3x^2$

$u'' = 16e^{4x}$ $\qquad$ $v'' = 6x$

$u''' = 64e^{4x}$ $\qquad$ $v''' = 6$

$u'''' = 256e^{4x}$

$u^{v} = 1024e^{4x}$

$y^5 = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x})6$
$+ 5(4e^{4x}) + e^{4x}$

$= 1024x^3e^{4x} + 3840x^2 \cdot e^{4x} + 3840xe^{4x} + 960e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

(3) If $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Soln

The eqn can be written as $x^2y'' + xy' + y = 0$

$w_1 = x^2y''$, $w_2 = xy'$ and $w_3 = y$

$w_1 = x^2y''$

let $u = y''$ $\quad$ $u^n = y^{n+2}$

$v = x^2$ $\quad$ $v' = 2x$ $\quad$ $v'' = 2$, $v''' = 0$

$w_1 = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2}y^n \times 2 + 0$

$w_2 = xy'$

let $u = y'$ $\quad$ $u^n = y^n$

$v = x$ $\quad$ $v' = 1$ $\quad$ $v'' = 0$

$w_2 = y^{(n+1)} \cdot x + ny^n \cdot 1 + 0$

$= xy^{n+1} + ny^n$

$w_3 = y^{(n)}$

$w = w_1 + w_2 + w_3$

$= x^2y^{(n+2)} + ny^{(n+1)} \cdot 2x + \frac{n(n-1)}{2}y^n \times 2 + xy^{(n+1)}x + ny^n$
$+ y^n = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n[n(n-1) + n + 1] = 0$

$x^2y^{(n+2)} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$