

# UNKNOWN THESE NOTES

LS/FN408/029

Mechanical Engg

1) If  $y = e^{x^2+x}$

Show that

$$y'' = y'(2x+1) + 2y$$

$$y = e^{x^2+x}$$

$$\frac{dy}{dx} = y'$$

$$\text{let } y = e^u \quad \frac{dy}{dx} = y' = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = y' = e^u \times 2x+1$$

$$y' = (2x+1)e^u$$

$$y' = (2x+1)e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = y''$$

$$y' = (2x+1)e^{x^2+x}$$

Using product rule

$$\text{let } u = 2x+1$$

$$v = e^{x^2+x}$$

$$\frac{du}{dx} = 2$$

$$\frac{dv}{dx} = 2x + e^{x^2+x}$$

$$\therefore \frac{d^2y}{dx^2} = y'' = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$= 2(e^{x^2+x}) + (2x+1)(2x+1)e^{x^2+x}$$

Since  $y = e^{x^2+x}$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2(y) + (2x+1)(y')$$

$$y'' = 2y + (2x+1)y'$$

$$y'' = y'(2x+1) + 2y$$

b) Hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

$$y'' = y'(2x+1) + 2y$$

$$y'' - y'(2x+1) - 2y = 0$$

Using Leibniz's theorem

1)  $y''$

$$u = y'' \quad v = 1$$

$$u'' = y^{n+2} \quad v' = 0$$

$$= u''v + n u^{n-1} u'$$

$$= y^{n+2}(1) + n(y^{n+1})(0)$$

$$= y^{n+2}$$

2)  $-y \cdot (2x+1)$

$$u = y'$$

$$v = -(2x+1)$$

$$u'' = y^{n+1}$$

$$v' = -2$$

$$v'' = 0$$

$$= u''v + n u^{n-1} v' + n(n-1) u^{n-2} v''$$

$$= -y^{n+1}(2x+1) + n(y^n)(-2)$$

$$= -(2x+1)y^{n+1} - 2ny^n$$

$$-2y$$

$$u = y$$

$$v = -2$$

$$u^n = y^n$$

$$v' = 0$$

$$= u^n v + n u^{n-1} v'$$

$$= -2y^n$$

$$\therefore y^{n+2} = -(2n+1)y^{n+1} - 2ny^n - 2y^n = 0$$

$$y^{n+2} = (2n+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2n+1)y^{(n+1)} + 2y^{n(n+1)}$$

$$y^{(n+2)} = (2n+1)y^{(n+1)} + 2(n+1)y^n$$

Q) Using the Leibnitz Theorem, given that

$$y = x^3 e^{4x}$$

determine  $y^{(5)}$

$$v = x^3$$

$$u = e^{4x}$$

$$v' = 3x^2$$

$$u' = 4e^{4x}$$

$$v^{(2)} = 6x$$

$$u^{(2)} = 16e^{4x}$$

$$v^{(3)} = 6$$

$$u^{(3)} = 64e^{4x}$$

$$v^{(4)} = 0$$

$$u^{(4)} = 256e^{4x}$$

$$u^{(5)} = 1024e^{4x}$$

$$y^3 = u^3 v + \frac{n(n-1)}{2!} u^2 v' + \frac{n(n-1)(n-2)}{3!} u v'^2 +$$

$$\frac{n(n-1)(n-2)(n-3)}{4!} u v^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u v^5$$

$$y^5 = [1024e^{4x}(x^3)] + [5(256e^{4x})3x^2] + \left[\frac{2n+1}{2} \cdot 64e^{4x} \times 6x\right] + \left[\frac{3n+1}{3!} \times 16e^{4x} \times 5\right] + [6][0]$$

$$y^5 = 1024e^{4x} x^3 + 3840e^{4x} (3x^2) + 640e^{4x} (6x) + 1600e^{4x} (5)$$

$$y^5 = 1024e^{4x} x^3 + 3840e^{4x} x^2 + 3840e^{4x} x + 9600e^{4x}$$

$$y^5 = e^{4x} (1024x^3 + 3840x^2 + 3840x + 9600)$$

$$u = y''$$

$$u^n = y^{n+2}$$

$$y^n = u^n u + n u^{n-1} v' + n(n-1) u^{n-2} v'' + n(n-1)(n-2) u^{n-3} v''' + \dots$$

$$y^n = y^{n+2} x^2 + n y^{n+1} (2x) + \frac{n(n-1)}{2!} y^n (2)^2 + \dots$$

$$y^n = y^{n+2} x^2 + 2x n y^{n+1} + n(n-1) y^n$$

iii)  $x y'$

$$u = y'$$

$$u^n = y^{n+1}$$

$$y^n = u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v'' + \dots$$

$$y^n = y^{n+1} x + n y^n (1) + \dots$$

$$y^n = x y^{n+1} + n y^n$$

3)  $y$

$$u = y \quad v = 1$$

$$u^n = y^n \quad v' = 0$$

$$y^n = u^n v + n u^{n-1} v'$$

$$= y^n (1) + n (y^{n-1}) (0)$$

$$y^n = y^{n+2} x^2 + n y^{n+1} (2x) + (n(n-1)) y^n + 2x n y^{n+1} + n y^n + y^n$$

$$y^n = x^2 (y^{n+2}) + 2x n (y^{n+1}) + (n^2 - n) y^n + x (y^{n+1}) + n y^n + y^n$$

$$0 = (n^2 - n + n + 1) y^n + (2x n + x) y^{n+1} + x (y^{n+1}) + n y^n + y^n$$

$$0 = x^2 y^{n+2} + (2x n + x) y^{n+1} + (n^2 + 1) y^n$$

$$0 = x^2 y^{n+2} + (2n + 1) x y^{n+1} + (n^2 + 1) y^n$$

$$x^2 y^{n+2} + (2n + 1) x y^{n+1} + (n^2 + 1) y^n = 0$$