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DEPT: MECHATRONICS

1) $y = e^{x^2+x}$

Let $y = x^2 + x$

$\frac{dy}{dx} = 2x+1$

$y = e^u$

$\frac{dy}{dx} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} + \frac{du}{dx}$

$\frac{dy}{dx} = e^u \times 2x+1$

where $u = x^2+x$

$= e^{x^2+x} \cdot (2x+1) = y'$

if $y' = e^{x^2+x}(2x+1$

then $y'' = 2e^{x^2} e^{x^2+x}(2) + [e^{x^2+x} \cdot (2x+1)](2x+1)$

$y'' = 2e^{x^2} \cdot e^{x^2+x}(2) + (e^{x^2+x}(2x+1)](2x+1)$

but $y = e^{x^2+x}$

$y' = e^{x^2+x}(2x+1$

$y'' = 2(y) + y'(2x+1)$

Applying leibnitz theorem

$y^{(n+2)} = 2(n+1)y^n + y(n+1)(2x+1$

2) Recall from leibnitz theorem

$y^5 = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + v^5$

where $u = e^{4x}$ | $v = x^3$

$u' = 4e^{4x}$ | $v' = 3x^2$

$u'' = 16e^{4x}$ | $v'' = 6x$

$u''' = 64e^{4x}$ | $v''' = 6$

$u^{iv} = 256e^{4x}$

$u^v = 1024e^{4x}$

Therefore

$y^5 = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})6x + 10(16e^{4x})6.$

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$= 1024x^3 e^{4x} + 3840x^2 \cdot e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

(iii) The eqn can be written as.

$x^2y'' + xy' + y = 0$

Let $w = x^2y''$, $w_2 = xy'$ and $w_3 = y$

for $w_1 = x^2 y''$

Let $u = y''$, $u^n = y^{n+2}$

Let $v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$w' = y^{n+2} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2} y^n x^2 + 0$

$w_3 = y^n$

Combining

$w = w_1 + w_2 + w_3$

$w = x^2 y^{(n+2)} + n y^{(n+1)} 2x + \frac{n(n-1)}{2} y^n \times 2 + xy'' + y^{(n+1)} x + n y'' + y^n = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n(n-1) + (n+1)) = 0$

$= x^2 y^{(n+2)} + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$

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