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Mat. NO: 15/ENG04/013

① (i) $y = e^{x^2+x}$ --- ①

Taking the first derivative

$$\frac{dy}{dx} = y' = (2x+1)e^{x^2+1} \quad \text{---} \; (ii)$$

$$\frac{d^2y}{dx^2} = y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$$

$$y'' = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x} \quad \text{---} \; (iii)$$

Equating (iii) to (ii) and ①

$$(4x^2+4x+1)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2[e^{x^2+x}] \quad \text{---} \; (iv)$$

$$(4x^2+4x+1)e^{x^2+x} = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x} \quad \text{---} \; (v)$$

If we observe in eqn (iv)

$$y'' = y'(2x+1) + 2y.$$

(ii)

(ii) $y'' = y'(2x+1) + 2y$

Using Leibnitz theorem

$$-y'' + y'(2x+1) + 2y = 0$$

Let $W^0 = y''$

$W' = y'''$

$W^n = -y^{n+2}$

Also Le $P = y'(2x+1)$

$\therefore$ $u = y'$ & $v = 2x+1$

$u' = y''$ $\quad$ $v' = 2$

$u'' = y'''$ $\quad$ $v'' = 0$

$u^n = y^{n+1}$

$$p(n) = u^n v^0 + n u^{n-1} v' + n(n-1)u^{n-2} v''$$

$$= y^{n+1}(2x+1) + n y^{n.2} + 0$$

$$p(n) = (2x+1)y^{n+1} + 2ny^n$$

Let $z = 2y$

$z^n = 2y^n$

Adding; $W^n + P^n + Z^n$

$-y^{n+2} + (2x+1)y^{n+1} + 2ny^n + 2y^n = 0$

$\Rightarrow y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

(2)

$y = x^3 e^{4x}$

$u = e^{4x}$ & $v = x^3$

$u^{(1)} = 4e^{4x}$ $\quad v^{(1)} = 3x^2$

$u^{(2)} = 16e^{4x}$ $\quad v^{(11)} = 6x$

$u^{(3)} = 64e^{4x}$ $\quad v^{(3)} = 6$

$u^{(4)} = 256e^{4x}$ $\quad v^{(4)} = 0$

$u^{(5)} = 1024e^{4x}$

from Leibnitz theorem

$$y^n = \cancel{\ } u^n v^0 + n\,u^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}v^{(11)} + n(n-1)(n-2)u^{(n-3)}v^{(3)}$$

$$+ \frac{n(n-1)(n-2)(n-3)\,u^{(n-4)}}{4!}v^{(4)}$$

Comparing $n = 5$,

$$y^{(5)} = u^{(5)}v^0 + 5u^{(4)}v' + \frac{(5\times4)\,u^{(3)}}{2}v^{(11)} + \frac{(5\times4\times3)\,u^{(2)}v^{(3)}}{6}$$

$$+ \frac{(5\times4\times3\times2)\,u'}{8}\cdot v^{(4)}$$

$y^{(5)} = 1024e^{4x}\cdot x^3 + 5(256e^{4x})3x^2 + 10(64e^{4x})6x + 10(16e^{4x}).6 + 0$

$y^{(5)} = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$

(11) $x^2y^{(11)} + xy^{(1)} + y = 0$

Let $W = x^2y^{(11)}$

So that $U = y^{(11)}$ & $V = x^2$

$U' = y^{(111)}$ $\quad V' = 2x$

$U'' = y^{(IV)}$ $\quad V'' = 2$

$U^n = y^{(n+2)}$ $\quad\quad V^{(iii)} = 0$

$W^{(n)} = U^n V^0 + nU^{(n-1)}V^{(i)} + \frac{n(n-1)}{2!}U^{(n-2)}V^{(ii)} + n(n-1)(n-2)U^{n-3}V^{(iii)}$

$= y^{(n+2)} \cdot x^2 + ny^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} y^n \cdot 2 + 0$

$W^{(n)} = x^2 y^{(n+2)} + n2xy^{(n+1)} + n(n-1)y^n$

Also $W = xy'$

So that $U = y'$ & $V = x$

$U' = y''$ $\quad V' = 1$

$U^n = y^{(n+1)}$ $\quad V'' = 0$

$W^{(n)} = U^n V^0 + nU^{(n-1)}V^{(i)} + \frac{n(n-1)}{2!}U^{(n-2)}V^{(ii)}$

$W^{(n)} = \cancel{xy}\ y^{(n+1)} \cdot x + ny^n + 0$

$W^{(n)} = xy^{(n+1)} + ny^n$

Also; $W = y$

$W^{(n)} = y^{(n)}$

Adding $x^2 y^{(n+2)} + n2xy^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$\Rightarrow x^2 y^{(n+2)} + (2n+1)y^{(n+1)} + (n^2 - n + n + 1)y^n = 0$

$x^2 y^{(n+2)} + (2n+1)y^{(n+1)} + (n^2+1)y^n = 0$