Arhorhire oghenomega Jephthah
MIN: 15/ENG05/001
Dept : Mechatronics

1) $y = e^{x^2+x}$

let $u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = e^u \times 2x + 1$

where $u = x^2 + x$

$= e^{x^2+x} \cdot (2x+1) = y'$

if $y' = e^{x^2+x}(2x+1)$

then $y'' = 2e^{x^2} \cdot e^{x^2+x}(2) + (e^{x^2+x} \cdot (2x+1))$
$(2x+1)$

$y'' = 2e^{x^2} \cdot e^{x^2+x}(2) + (e^{x^2+x}(2x+1))(2x+1)$

but $y = e^{x^2+x}$

$y' = e^{x^2+x}(2x+1)$

$y'' = 2(y) + y'(2x+1)$

Applying Leibnitz theorem

$y(n+2) = 2(n+1)y^n + y(n+1)(2x+1)$

2) Recall from Leibnitz theorem

$y^5 = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + v^5$

where $u = e^{4x}$ $v = x^3$

$u' = 4e^{4x}$ $v' = 3x^2$

$u'' = 16e^{4x}$ $v'' = 6x$

$u''' = 64e^{4x}$ $v'' = 6x$

$u^{iv} = 256e^{4x}$

$u^{v} = 1024e^{4x}$

$= 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

(iii) The eqn can be written as

$x^2 y'' + xy' + y = 0$

Let $w = x^2 y''$, $w_2 = xy'$ and $w_3 = y$

Now $w_1 = x^2 y''$

Let $u = y^2$ and $v = y^{n+1}$

Let $w = x^2$ $w' = 2x$ $w'' = 2$

$w' = y^{n+2} x^2 + ny^{(n+1)} 2x + \frac{n(n-1)}{2} y^{n-2} \cdot 8$

$w_3 = y^n$