1) If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and, hence, prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$y = e^{x^2+x}$

$\frac{dy}{dx} = (2x+1)e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= e^{x^2+x}[(2x+1)(2x+1)+2] \cdot y'(2x+1)+2y$

$= (2x+1)e^{x^2+x}(2x+1) + 2y \cdot (2x+1)e^{x^2+x}$

$(2x+1) + 2e^{x^2+x}$

$d^2y/dx^2 = e^{x^2+x}[(2x+1)(2x+1)+2]$

$y'' = y'(2x+1) + 2y$

Let d_1, d_2 & d_3 represent y'', y' & y

d_1, $v = y^2$

$v^n = y^{(n+2)}$

d_2, $u = y'$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$

$u^{n-1} = y^n$ $\quad v'' = 0$

d_3, $u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v' = 0$

Using $d_1 = d_2 + d_3$

$y^{n+2} = y^{n+1}(2x+1) + ny^n \cdot 2 + y^n 2$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) Using the Leibnitz theorem, given that

a) $y = x^3e^{4x}$, determine $y^{(5)}$

b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, ~~show that~~

Show that,

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$.

Soln

a) $y = x^3 e^{4x}$ $\qquad V = x^3$

$u = e^{4x}$ $\qquad V' = 3x^2$

$u^n = 4^n e^{4x}$ $\qquad V'' = 6x$

$u^{n-1} = 4^{(n-1)} e^{4x}$ $\qquad V''' = 6$

$u^{n-2} = 4^{(n-2)} e^{4x}$ $\qquad V^{iv} = 0$

$u^{n-3} = 4^{(n-3)} e^{4x}$

$$y^n = 4^n e^{4x} x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{(n-1)} e^{4x} \cdot 6x$$

$$+ \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} \cdot 6$$

When $n = 5$

$$y^{(5)} = 4^5 e^{4x} x^3 + (5)(4)^{(5-1)} e^{4x} \cdot 3x^2 + \frac{5(5-1)}{2!} 4^{(5-1)} e^{4x}$$

$$\cdot 6x + \frac{5(5-1)(5-2)}{3!} 4^{5-3} e^{4x} \cdot 6$$

$$y^{(5)} = 4^5 e^{4x} x^3 + 20^4 e^{4x} \cdot 3x^2 + (10) 256 e^{4x} \cdot 6x$$
$$+ (10)(16) e^{4x} \cdot 6$$

$$y^{(5)} = 4^5 e^{4x} x^3 + 20^4 e^{4x} \cdot 3x^2 + 2560 e^{4x} \cdot 6x + 160 e^{4x}$$
$$\cdot 6,$$

b) $x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$

$x^2 (y^{2}) + x y^{(1)} + y = 0$

Let d_1, d_2 & d_3 represent y^{2}, y' & y

d_1,

$u = y^{(2)}$ $\qquad v = x^2$

$u^n = y^{n+2}$ $\qquad v' = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\qquad v'' = 2$

$u^{(n-2)} = y^{(n)}$ $\qquad v''' = 0$

d_2,

$u = y^{(1)}$ $\qquad v = x$

$u^n = y^{n+1}$ $\qquad v' = 1$

$u^{n-1} = y^n$ $\qquad v'' = 0$

d_3,

$u = y$

$u^n = y^n$

using

$d_1 + d_2 + d_3 = 0$

$y^{n+2} \cdot x^2 + n y^{n+1} \cdot 2x + \frac{n(n-1)}{2!} y^n \cdot 2 + y^{n+1} \cdot x + n y^n$

$+ y^n = 0$

$x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n-1) y^n + x y^{n+1} + n y^n + y^n$

$= 0$

$x^2 y^{(n+1)} + 2x n y^{(n+1)} + x y^{(n+1)} + n(n-1) y^n + n y^n + y^n$

$= 0$

$x^2 y^{(n+1)} + x y^{(n+1)} (2n+1) + y^n [n(n-1) + (n+1)] = 0$

$\cancel{x^2 y^{(n+1)} + x y^{(n+1)} (2n+1) + y^n}$

$x^2 y^{(n+1)} + x y^{(n+1)} (2n+1) + y^n (n^2+1) = 0$

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