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15/ENG03/027
CIVIL ENGINEERING

1) If $y = e^{x^2+x}$

let $u = x^2+x$

$\frac{du}{dx} = 2x+1$

$y = e^u$

$dy/du = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = e^u \times 2x+1$

where $u = x^2+x$

$\frac{dy}{dx} = e^{x^2+x}.(2x+1) = y'$

if $y' = e^{x^2+x}(2x+1)$

Then

$y'' = e^{x^2+x}.(2) + [e^{x^2+x}.(2x+1)](2x+1)$

$y'' = 2.e^{x^2+x} + e^{x^2+x}.(2x+1).(2x+1)$

but $y = e^{x^2+x}$

and $y' = e^{x^2+x}(2x+1)$

$y'' = 2(y) + y'(2x+1)$

Applying leibnitz theorem

$y^{(n+2)} = 2(n+1)y^n + y^{(n+1)}(2x+1)$

2) $y = x^3e^{4x}$

using leibnitz theorem

$y = u^5v + 5u^4v' + 10u^3v^2 + 10u^2v^3 + 5u'v^4 + v^5$

when $u = e^{4x}$ $\quad v = x^3 \quad v' = 3x^2 \quad v'' = 6x$

$u' = 4e^{4x}$ $\quad v''' = 6$

$u'' = 16e^{4x}$

$u''' = 64e^{4x}$

$u^{iv} = 256e^{4x}$

$u^{v} = 1024e^{4x}$

$y^{(5)} = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x})(6) + 5(4e^{4x})(0) + (0)^5$

$y^{(5)} = 1024x^3 \cdot e^{4x} + 3840x^2 \cdot e^{4x} + 3840x \cdot e^{4x} + 960e^{4x}$

$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

If $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $\cancel{x^2 y^{(n+2)}}$ $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

solution

$x^2y'' + xy' + y = 0$

~~let $w_1 = x^2y''$~~ , ~~$w = xy' = w$.~~

for $w_1 = x^2y''$

$v = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$

$u = y^{(2)} \quad u^{(n)} = y^{(n+2)} \quad u^{(n-1)} = y^{(n+1)} \quad u^{(n-2)} = y^{(n)}$

$w_1 = y^{(n+2)} \cdot x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^n \cdot 2 + 0$

for $w_2 = xy'$

$w_2 = y^{(n+1)} \cdot x + ny^{(n)} \times 1 + 0$

for $w_3 = y^{(n)}$

$x^2y^{(n+2)} + ny^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} \times 2 \times y^n + xy' + y^{(n+1)} \cdot 3 + ny^{(n)} + y^{(n)} = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n(n-1) + n + 1) = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 - 0 + 1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

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