

**COLLEGE OF ENGINEERING**

**DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING**

**PROCESS DYNAMICS & CONTROL**

**CHE 531 ASSIGNMENT II**

**BY**

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QUESTION 1



Mass Balance

$\frac{dm}{dt}=Ṁ\_{in}-Ṁ\_{out}$ -----------------------------------(1a)

$ρ=\frac{m}{V}$ $m=ρV$ $Ṁ=ρF$

$\frac{d(ρV)}{dt}=ρ\_{in}F\_{in}-ρ\_{out}F\_{out}$-------------------------(1b)

$ρ\frac{dV}{dt}=ρ\left(F\_{in}-F\_{out}\right)$-------------------------------(1c)

$\frac{dV}{dt}=F\_{in}-F\_{out}$--------------------------------------(2)

$F=β\sqrt{h}$ $V=Ah$

$\frac{d(Ah)}{dt}=F\_{in}-β\sqrt{h}$---------------------------------(3a)

$A\frac{dh}{dt}=F\_{in}-β\sqrt{h}$----------------------------------(3b)

$A\frac{dh}{dt}+β\sqrt{h}=F\_{in}$-----------------------------------(4)

$β\sqrt{h} $ is a non-linear term. To develop the linearized approximation for the non-linear model, the Taylor series expansion of the term, $β\sqrt{h}$, around a point $h\_{0}$ will be taken:

$$β\sqrt{h}=β\sqrt{h\_{0}}+\left[\frac{d}{dh}β\sqrt{h}\right]\_{h=h\_{0}}\left(h-h\_{0}\right)+\left[\frac{d^{2}}{dh^{2}}β\sqrt{h}\right]\_{h=h\_{0}}\frac{\left(h-h\_{0}\right)^{2}}{2!}+…$$

$$=β\sqrt{h\_{0}}+\frac{β}{2\sqrt{h\_{0}}}\left(h-h\_{0}\right)-\frac{β^{2}}{8^{3}\sqrt{h\_{0}^{2}}}\left(h-h\_{0}\right)^{2}+…$$

Neglecting terms of order two and higher. We’ll have:

$β\sqrt{h}≈β\sqrt{h\_{0}}+\frac{β}{2\sqrt{h\_{0}}}\left(h-h\_{0}\right)$-----------(\*)

Substituting equation (\*) in equation (4) will give

$A\frac{dh}{dt}+\frac{β}{2\sqrt{h\_{s}}}h=F\_{i}-\frac{β}{2}\sqrt{h\_{s}}$-------------------(5)

The equation above is the dynamic state equation. At steady state, the equation will be:

$A\frac{dh\_{s}}{dt}+\frac{β}{2\sqrt{h\_{s}}}h\_{s}=F\_{is}-\frac{β}{2}\sqrt{h\_{s}}$-------------------(6)

Subtracting equation 6 from 5

$A\frac{d(h-h\_{s})}{dt}+\frac{β}{2\sqrt{h\_{s}}}(h-h\_{s})=(F\_{i}-F\_{is})$-------------------(7)

But $h^{'}=h-h\_{s}$ $F^{'}=F\_{i}-F\_{is}$, then equation (7) will become

$A\frac{dh^{'}}{dt}+\frac{β}{2\sqrt{h\_{s}}}h^{'}=F^{'}$-------------------(8)

Dividing through by $\frac{β}{2\sqrt{h\_{s}}}$ will give

$\frac{2A\sqrt{h\_{s}}}{β}\frac{dh^{'}}{dt}+h^{'}=\frac{2\sqrt{h\_{s}}}{β}F^{'}$--------------------(9)

Let $τ\_{p}=\frac{2A\sqrt{h\_{s}}}{β}$ and $K\_{p}=\frac{2\sqrt{h\_{s}}}{β}$

Then equation (9) will be

$τ\_{p}\frac{dh^{'}}{dt}+h^{'}= K\_{p}F^{'}$----------------(10)

To find the transfer function, we’ll have to get the Laplace of the system:

$$y=y(s)$$

$$y^{'}\left(t\right)=sy\left(s\right)-y\left(0\right)$$

$τ\_{p}[s\overbar{h}\left(s\right)-\overbar{h}\left(0\right)]+\overbar{h}\left(s\right)= K\_{p}\overbar{F}\left(s\right)$----------------(11)

$$h=h\_{s}, \overbar{h}\left(0\right)=0$$

$τ\_{p}s\overbar{h}\left(s\right)+\overbar{h}\left(s\right)= K\_{p}\overbar{F}\left(s\right) $-------------------------------12

$\overbar{h}\left(s\right)\left(τ\_{p}s+1\right)=K\_{p}\overbar{F}\left(s\right) $--------------------------------------12a

$G\left(s\right)=\frac{\overbar{h}\left(s\right)}{\overbar{F}\left(s\right)}=\frac{K\_{p}}{\left(τ\_{p}s+1\right)}$----------------------------------------13

Equation 13 is the transfer function equation.

$$A=2.5m^{2} h\_{s}=4m β=2\frac{m^{\frac{5}{2}}}{min}$$

$$τ\_{p}=\frac{2A\sqrt{h\_{s}}}{β}=\frac{2\*2.5\*\sqrt{4}}{2}=5$$

and

$$K\_{p}=\frac{2\sqrt{h\_{s}}}{β}=\frac{2\*\sqrt{4}}{2}=2$$

$$G\left(s\right)=\frac{\overbar{h}\left(s\right)}{\overbar{F}\left(s\right)}=\frac{K\_{p}}{\left(τ\_{p}s+1\right)}=\frac{2}{5s+1}$$

To solve manually:

$$G\left(s\right)=\frac{\overbar{h}\left(s\right)}{\overbar{F}\left(s\right)}=\frac{K\_{p}}{\left(τ\_{p}s+1\right)}=\frac{2}{5s+1}$$

$$\overbar{h}\left(s\right)=\frac{2}{5s+1}\overbar{F}\left(s\right)$$

Applying a unit step change in the manipulated variable:

$$\overbar{h}\left(s\right)=\frac{2}{5s+1}\*\frac{1}{s}=\frac{2}{s\left(5s+1\right)}$$

Solving by Partial fraction

$$\overbar{h}\left(s\right)=\frac{2}{s\left(5s+1\right)}=\frac{A}{s}+\frac{B}{5s+1}$$

$$2=A\left(5s+1\right)+Bs$$

Let $s=0$

$$2=A\left(5(0)+1\right)+B(0)$$

$$A=2$$

Let $s=-\frac{1}{5}$

$$2=A\left(5(-\frac{1}{5})+1\right)+B(-\frac{1}{5})$$

$$B=-10$$

$$\overbar{h}\left(s\right)=\frac{2}{s\left(5s+1\right)}=\frac{2}{s}-\frac{10}{5s+1}$$

Taking the inverse Laplace of $\overbar{h}\left(s\right)$ will give:

$$\overbar{h}\left(t\right)=L^{-1}\left\{\overbar{h}\left(s\right)\right\}=L^{-1}\left\{\frac{2}{s}-\frac{10}{5s+1}\right\}$$

$$\overbar{h}\left(t\right)=L^{-1}\left\{\frac{2}{s}\right\}-L^{-1}\left\{\frac{10}{5s+1}\right\}$$

$$\overbar{h}\left(t\right)=L^{-1}\left\{\frac{2}{s}\right\}-2\*L^{-1}\left\{\frac{1}{s+\frac{1}{5}}\right\}$$

$$\overbar{h}\left(t\right)=2-2e^{-\frac{1}{5}t}$$

$$\overbar{h}\left(t\right)=2(1-e^{-\frac{1}{5}t})$$

$At t=1min, h(t)= 0.362538$, the model reaches steady state at t=30mins

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**QUESTION 2**

$$G\_{p}\left(s\right)=\frac{2}{5s+1}$$

$$G\_{f}\left(s\right)=G\_{m}\left(s\right)=1, k\_{c}=1.5$$

Set point method:

$$y\left(s\right)=\frac{G\_{p}\left(s\right)G\_{f}\left(s\right)G\_{c}\left(s\right)}{1+G\_{p}\left(s\right)G\_{f}\left(s\right)G\_{c}\left(s\right)G\_{m}\left(s\right)}Y\_{sp}\left(s\right)$$

$$y\left(s\right)=\frac{\frac{2}{5s+1}\*1.5}{1+\frac{2}{5s+1}\*1.5}\*\frac{3}{s}$$

$$y\left(s\right)=\frac{\frac{3}{5s+1}}{1+\frac{3}{5s+1}}\*\frac{3}{s}=\frac{3}{5s+4}\*\frac{3}{s}=\frac{9}{s(5s+4)}$$

Solving by Partial fraction

$$y\left(s\right)=\frac{9}{s\left(5s+4\right)}=\frac{A}{s}+\frac{B}{5s+4}$$

$$9=A\left(5s+4\right)+Bs$$

Let $s=0$

$$9=A\left(5(0)+4\right)+B(0)$$

$$A=\frac{9}{4}$$

Let $s=-\frac{4}{5}$

$$9=A\left(5(-\frac{4}{5})+4\right)+B(-\frac{4}{5})$$

$$B=-\frac{45}{4}$$

$$y\left(s\right)=\frac{9}{s\left(5s+4\right)}=\frac{9}{4}\*\frac{1}{s}-\frac{45}{4}\left(\frac{1}{5s+4}\right)$$

$$y\left(s\right)=\frac{9}{4}\left(\frac{1}{s}-\frac{5}{5s+4}\right)$$

$$y\left(s\right)=\frac{9}{4}\left(\frac{1}{s}-\frac{1}{s+\frac{4}{5}}\right)$$

Taking the inverse Laplace of $\overbar{h}\left(s\right)$ will give:

$$y\left(t\right)=L^{-1}\left\{y\left(s\right)\right\}=\frac{9}{4}\left[L^{-1}\left\{\frac{1}{s}-\frac{1}{s+\frac{4}{5}}\right\}\right]$$

$$y\left(t\right)=\frac{9}{4}\left[L^{-1}\left\{\frac{1}{s}\right\}-L^{-1}\left\{\frac{1}{s+\frac{4}{5}}\right\}\right]$$

$$y\left(t\right)=$$

$$y\left(t\right)=\frac{9}{4}(1-e^{-\frac{4}{5}t})$$

