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Chemical Engineering
14/ENG01/020
CHE 531
Assignment

2) $G_p(s) = \dfrac{2}{5s+1}$

$G_f(s) = G_m(s) = 1$

3 unit step change

$K_c = 1.5$ for a p-only controller

Using this formula:

$$y(s) = \frac{G_p(s)\,G_f(s)\,G_c(s)}{1 + G_p(s)\,G_f(s)\,G_c(s)\,G_m(s)} \quad y_{sp}(s)$$

$$y_{sp}(s) = \frac{3}{s}$$

$$y(s) = \frac{\frac{2}{5s+1} \times 1 \times 1.5}{1 + \frac{2}{5s+1} \times 1 \times 1 \times 1.5} \cdot \frac{3}{s}$$

$$= \cancel{\frac{2}{5s+1}} = \cancel{\frac{\frac{2}{5s+1}}{1 + \frac{2}{5s+1}}}$$

$$= \frac{\frac{3}{5s+1}}{1 + \frac{3}{5s+1}} \cdot \frac{3}{s} = \frac{\frac{3}{5s+1}}{\frac{5s+1+3}{5s+1}} \cdot \frac{3}{s}$$

$$= \frac{3}{\cancel{5s+1}} \times \frac{\cancel{5s+1}}{5s+4} \times \frac{3}{s}$$

$$y(s) = \frac{9}{s(5s+4)} = \frac{A}{s} + \frac{B}{5s+4}$$

$$y(s) = \frac{A(5s+4) + B(s)}{s(5s+4)}$$

$$9 = A(5s+4) + B(s)$$

Let $s = 0$

$$9 = A(5(0)+4) + B(0)$$

$$9 = A(0+4) + 0$$

$$9 = 4A$$

$$A = 9/4$$

Let $s = -4/5$

$$9 = A\left(5(-4/5)+4\right) + B(-4/5)$$

$$9 = A(-4+4) - \frac{4}{5}B$$

$$9 = A(0) - \frac{4}{5}B$$

$$45 = -4B$$

$$B = -\frac{45}{4}$$

$$y(s) = 9/4\,(1/s) - 45/4\left[\frac{1}{5s+4}\right]$$

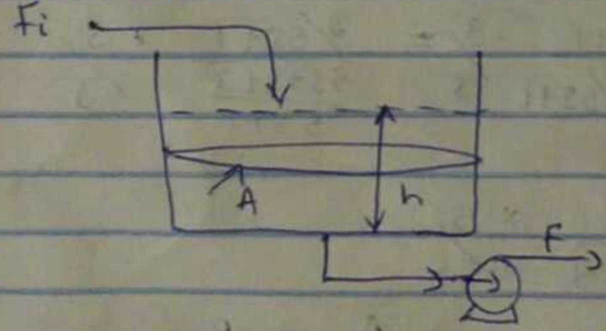
$$= \frac{9}{4}\left[\frac{1}{s} - \frac{5}{5s+4}\right] = \frac{9}{4}\left[\frac{1}{s} - \frac{5/5}{5s/5+4/5}\right]$$

$$y(s) = \frac{9}{4}\left[\frac{1}{s} - \frac{1}{s+4/5}\right]$$

$$y(t) = \mathcal{L}^{-1}\,y(s)$$

$$y(t) = \frac{9}{4}\left[1 - e^{-4/5\,t}\right]$$

(1)



and $F = \beta\sqrt{h}$

a) $$\frac{dv}{dt} = F_i - F$$

$$V = Ah$$

$$\frac{d(Ah)}{dt} = F_i - F$$

$$A\frac{dh}{dt} = F_i - \beta\sqrt{h}$$

$$A\,dh/dt + \beta\sqrt{h} = F_i$$

Using Taylor series to Linearize

$$\beta\sqrt{h} = \beta\sqrt{h_0} + \left[\frac{d}{dh}(\beta\sqrt{h})\right]_{h=h_0}(h-h_0) + \left[\frac{d^2}{dh^2}(\beta\sqrt{h})\right]_{h=h_0}\frac{(h-h_0)^2}{2!} + \text{-----}$$

$$= \beta\sqrt{h_0} + \frac{\beta}{2\sqrt{h_0}}(h-h_0) - \frac{\beta}{8\sqrt{h_0^3}}(h-h_0)^2 + \text{----}$$

$$\beta\sqrt{h} \approx \beta\sqrt{h_0} + \frac{\beta}{2\sqrt{h_0}}(h-h_0)$$

$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_0}}h = F_i - \frac{\beta}{2}\sqrt{h_0}$$ — dynamic model

b) $$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_s}}h = F_i - \frac{\beta}{2}\sqrt{h_s}$$ - steady state

$$A\frac{d(h-h_s)}{dt} + \frac{\beta}{2\sqrt{h_s}}(h-h_s) = F_i - F_{is}$$

$$A\frac{d\bar{h}}{dt} + \frac{\beta}{2\sqrt{h_s}}\bar{h} = \bar{F_i}$$

dividing through by $\beta/2\sqrt{h_s}$

$$\frac{2A\sqrt{h_s}}{\beta}\frac{dh}{dt} + \bar{h} = \frac{2\sqrt{h_s}}{\beta}\bar{F_i}$$

let $\tau_p = \dfrac{2A\sqrt{h_s}}{\beta}$ and $k_p = \dfrac{2\sqrt{h_s}}{\beta}$

$$\tau_p\frac{dh}{dt} + \bar{h} = k_p\bar{F_i}$$

To get the transfer function

$$\tau_p[s\bar{h}(s) - \bar{h}(0)] + \bar{h}(s) = k_p F_i(s)$$

$$\tau_p s h(s) + \bar{h}(s) = k_p F_i(s)$$

~~$h(s)[\tau_p s$~~

$$h(s)[\tau_p s + 1] = k_p F_i(s)$$

$$G_p(s) = \frac{k_p}{\tau_p s + 1} = \frac{\bar{h}(s)}{F_i(s)}$$

and $\tau_p = \dfrac{2A\sqrt{h_s}}{\beta}$

$A = 2.5\,m^2$ $\quad h = 4m$

$\beta = 2\,\frac{m^{5/2}}{min}$

$$= \frac{2\times 2.5\times\sqrt{4}}{2} = 5$$

$$K_p = \frac{2\sqrt{h_s}}{\beta}$$

$$= \frac{2\sqrt{4}}{2}$$

$$= 2$$

$$G(s) = \frac{K_p}{\tau_p s + 1} = \frac{2}{5s+1}$$

Applying a unit step change ($1/s$)

$$\bar{h}(s) = \frac{2}{5s+1} \cdot \frac{1}{s} = \frac{2}{s(5s+1)}$$

$$\bar{h}(s) = \frac{A}{s} + \frac{B}{5s+1} = \frac{A(5s+1) + B(s)}{s(5s+1)}$$

$$2 = A(5s+1) + B(s)$$

Let $s = 0$

$$2 = A(5(0)+1) + B(0)$$

$$2 = A(1) + 0$$

$$2 = A$$

Let $s = -1/5$

$$2 = A(5(-1/5)+1) + B(-1/5)$$

$$2 = A(-1+1) - B/5$$

$$2 = 0 - B/5$$

$$10 = -B, \quad B = -10$$

$$h(s) = 2(1/s) - 10\left(\frac{1}{5s+1}\right) = 2/s - \frac{10/5}{5s/5 + 1/5}$$

$$h(t) = \mathcal{L}^{-1} h(s)$$

$$h(t) = 2 - 2e^{-t/5}$$

$$= 2[1 - e^{-t/5}]$$