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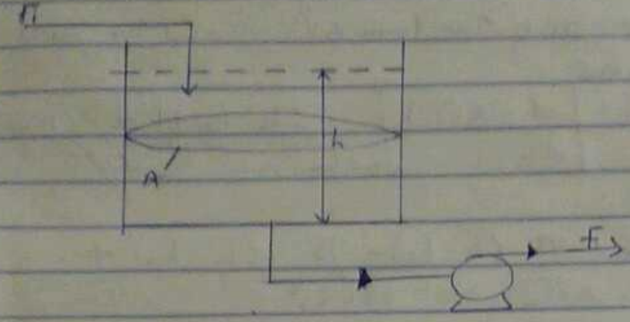
COURSE TITLE: Process Dynamics and Control (CHE 531)

(1)



Considering the mass balance of this system.

$$\frac{dm}{dt} = \dot{M}_{in} - \dot{M}_{out}$$

$$\rho = m/v$$

$$m = \rho v$$

$$\dot{m} = \rho f$$

f = volumetric flowrate.

$$\frac{d(\rho v)}{dt} = \rho_{in} f_{in} - \rho_{out} f_{out}$$

If density is constant

$$\rho \frac{dv}{dt} = \rho f_{in} - \rho f_{out}$$

$$\frac{dv}{dt} = f_{in} - f_{out}$$

let $f_{out} = f$

$$\frac{dv}{dt} = f_{in} - f$$

But $v = Ah$

$$\frac{dAh}{dt} = f_{in} - f$$

$$A\frac{dh}{dt} = f_i - f$$

but $\beta = \frac{k}{R}$ from the question

$$f = \beta\sqrt{h}$$

$$A\frac{dh}{dt} = f_{in} - \beta\sqrt{h} \quad \text{——— (1)}$$

$\beta\sqrt{h}$ is the only non-linear term, to develop the linearized approximation for this nonlinear model, take the Taylor series expansion of the term $\beta\sqrt{h}$ around the point h_0. Therefore it is given as

$$\beta\sqrt{h} = \beta\sqrt{h_0} + \left[\frac{d(\beta\sqrt{h})}{dh}\right]_{h=h_0}(h-h_0) + \left[\frac{d^2(\beta\sqrt{h})}{dh^2}\right]_{h=h_0}\frac{(h-h_0)^2}{2!}$$

$$+ \dots$$

$$= \beta\sqrt{h_0} + \frac{\beta}{2\sqrt{h_0}}(h-h_0) - \frac{\beta}{8\sqrt[3]{h_0^2}}(h-h_0)^2 + \dots$$

Neglecting terms in order two and higher, we take

$$\beta\sqrt{h} \approx \beta\sqrt{h_0} + \frac{\beta}{2\sqrt{h_0}}(h-h_0) \quad \text{——— (*)}$$

Putting equn (*) into equn (1)

$$A\frac{dh}{dt} - \beta\sqrt{h} = f_{in}$$

$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_0}}(h-h_0) = f_i - \frac{\beta}{2}\sqrt{h_0}$$

$\therefore$

(or) $$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_0}}h = f_i - \frac{\beta}{2}\sqrt{h_0} \quad \text{— Dynamic State Equation}$$

for steady state equation

$$A\frac{dh_s}{dt} + \frac{\beta}{2\sqrt{h_s}}h_s = f_{is} = \frac{\beta\sqrt{h_s}}{2}$$

Subtracting the steady state from the dynamic state. We have

$$A\frac{d(h-h_s)}{dt} + \frac{\beta}{2\sqrt{h_s}}(h-h_s) = (f_i - f_{is}) \quad \text{——— (2)}$$

$$\left.\begin{aligned} h' &= h - h_s \\ f' &= f_i - f_{is} \end{aligned}\right\} \text{equn (**)}$$

Put equn (**) into equn (2)

$$A\frac{dh'}{dt} + \frac{\beta}{2\sqrt{h_s}}h' = f'$$

Divide through by $\dfrac{\beta}{2\sqrt{h_s}}$

$$\frac{2A\sqrt{h_s}}{\beta}\frac{dh'}{dt} + h' = \frac{2\sqrt{h_s}}{\beta} f' \quad \text{———— (3)}$$

let $\tau_p = \dfrac{2A\sqrt{h_s}}{\beta}$

$k_p = \dfrac{2\sqrt{h_s}}{\beta}$

Putting in to eqn (3)

$$\tau_p \frac{dh'}{dt} + h' = k_p f' \quad \text{———— (4)}$$

b) finding the transfer function of the system, we get the laplace

$$y = y(s)$$

$$y'(t) = sy(s) - y(0)$$

Put into eqn (4)

$$\tau_p[s\bar{h}(s) - \bar{h}(0)] + \bar{h}(s) = k_p \bar{f}(s)$$

But $h = h_s$ and $\bar{h}(0) = 0$

$$\tau_p s\bar{h}(s) + \bar{h}(s) = k_p \bar{f}(s)$$

$$\bar{h}(s)(\tau_p s + 1) = k_p \bar{f}(s)$$

$$G(s) = \frac{\bar{h}(s)}{\bar{f}(s)} = \frac{k_p}{(\tau_p s + 1)}$$

from the question given $A = 2.5m^2$, $h_s = 4m$, $\beta = \dfrac{2\,m^{5/2}}{min}$

$$\tau_p = \frac{2A\sqrt{h_s}}{\beta}$$

$$= \frac{2 \times 2.5 \times \sqrt{4}}{2}$$

$$\tau_p = 5$$

$$k_p = \frac{2\sqrt{h_s}}{\beta}$$

$$= \frac{2 \times \sqrt{4}}{2}$$

$$k_p = 2$$

$$G(s) = \frac{\bar{h}(s)}{\bar{f}(s)} = \frac{K_p}{(\tau_p s+1)}$$
$$= \frac{2}{5s+1}$$

But $\bar{h}(s) = \frac{2}{5s+1} \bar{f}(s)$

Applying a unit step change, we have

$$\bar{h}(s) = \frac{2}{5s+1} \times \frac{1}{s} = \frac{2}{s(5s+1)}$$

Solving by partial fraction.

$$\bar{h}(s) = \frac{2}{s(5s+1)}$$
$$= \frac{A}{s} + \frac{B}{5s+1}$$

$$\frac{2}{s(5s+1)} = \frac{A(5s+1) + Bs}{s(5s+1)}$$
$$2 = A(5s+1) + Bs$$

Let $s = 0$

$$2 = A(5(0)+1) + B(0)$$
$$2 = A(0+1)$$
$$\therefore A = 2$$

Let $s = -1/5$

$$2 = A(5(-1/5)+1) + B(-1/5)$$
$$2 = A(-1+1) - B/5$$
$$B = -5 \times 2$$
$$B = 10$$

$$\bar{h}(s) = \frac{2}{s(5s+1)} = \frac{2}{s} - \frac{10}{5s+1}$$

Taking the inverse laplace

$$\bar{h}(t) = \mathcal{L}^{-1}\{\bar{h}(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s} - \frac{10}{5s+1}\right\}$$
$$\bar{h}(t) = \mathcal{L}^{-1}\left\{\frac{2}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{10}{5s+1}\right\}$$
$$\bar{h}(t) = \mathcal{L}^{-1}\left\{2/s\right\} - \mathcal{L}^{-1}\left\{\frac{2}{s+1/5}\right\}$$
$$\bar{h}(t) = 2 - 2e^{-1/5t}$$
$$h(t) = 2(1 - e^{-1/5t})$$

QUESTION 2

$$G_p(s) = \frac{2}{5s+1}$$

Given that $G_f(s) = G_m(s) = 1$ and $k_c = 1.5$

Using the set point method.

$$y(s) = \frac{G_p(s)G_f(s)G_c(s)}{1 + G_p(s)G_f(s)G_c(s)G_m(s)} Y_{sp}(s)$$

$$y(s) = \frac{\frac{2}{5s+1} \times 1.5}{1 + \frac{2}{5s+1} \times 1.5} \times \frac{3}{s}$$

$$y(s) = \frac{\frac{3}{5s+1}}{1 + \frac{3}{5s+1}} \times \frac{3}{s}$$

$$y(s) = \frac{3}{5s+4} \times \frac{3}{s}$$

$$y(s) = \frac{9}{s(5s+4)}$$

Solving by partial fraction

$$\frac{9}{s(5s+4)} = \frac{A}{s} + \frac{B}{5s+4}$$

$$9 = A(5s+4) + Bs$$

when $s = 0$

$$9 = A(5(0)+4) + B(0)$$

$$9 = A(0+4)$$

$$A4 = 9$$

$$A = 9/4$$

when $s = -4/5$

$$9 = A(5(-4/5)+4) + B(-4/5)$$

$$9 = A(-4+4) - B\,4/5$$

$$B\,4/5 = -9$$

$$B = -9 \div 4/5$$

$$B = -9 \times 5/4$$

$$B = -45/4$$

$$y(s) = \frac{9}{s(5s+4)} = \frac{9}{4} \times \frac{1}{s} - \frac{45}{4}\left(\frac{1}{5s+4}\right)$$

$$y(s) = \frac{9}{4}\left(\frac{1}{s} - \frac{5}{5s+4}\right)$$

$$y(s) = \frac{9}{4}\left(\frac{1}{s} - \frac{1}{s+4/5}\right)$$

Taking the inverse Laplace, we have

$$y(t) = \mathcal{L}^{-1}\{y(s)\} = \frac{9}{4}\left[\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+4/5}\right\}\right]$$

$$y(t) = \frac{9}{4}\left[\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left\{\frac{1}{s+4/5}\right\}\right]$$

$$y(t) = \frac{9}{4}\left(1 - e^{-4/5\,t}\right)$$