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15/ENG07/046
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1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$y'' + 4y' + 5y = 6\sin\theta$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = 5$

$k^2 + 4k + 2^2 = -5 + 2^2$

$(k+2)^2 = -1$

$(k+2) = \pm\sqrt{-1}$

$k + 2 = \pm i$

$k = \pm i - 2$

$\therefore k_1 = i - 2, \quad k_2 = -i - 2$

$y_1 = C_1 e^{(-2+i)\theta} + C_2 e^{(-2-i)\theta}$

$y_u = C_1 e^{-2\theta + i\theta} + C_2 e^{-2\theta - i\theta}$

$y_u = e^{-2\theta}[C_1 e^{i\theta} + C_2 e^{-i\theta}]$

$y_u = e^{-2\theta}[Ae^{i\theta} + Be^{-i\theta}]$

$y_u = e^{-2\theta}[A\cos\theta + B\sin\theta]$

$y_p = A\cos\theta + B\sin\theta$

$y_p' = -A\sin\theta + B\cos\theta$

$y_p'' = -A\cos\theta - B\sin\theta$

$y'' + 4y' + 5y = 6\sin\theta$

$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$(-A\cos\theta + 4B\cos\theta + 5A\cos\theta) + (-B\sin\theta - 4A\sin\theta + 5B\sin\theta) = 6\sin\theta$

$[-A + 4B + 5A]\cos\theta + [-B - 4A + 5B]\sin\theta = 6\sin\theta$

$4A + 4B = 0$

$-4A + 4B = 6$

$8B = 6$

$B = \frac{6}{8} = \frac{3}{4}$

Recall that $4A + 4B = 0$

$4A + 4(\frac{3}{4}) = 0$

$4A + 3 = 0$

$A = -3/4$

$\therefore y_p = -3/4 \cos\theta + 3/4 \sin\theta$

$y = y_c + y_p$

$y = e^{-2\theta}[A\cos\theta + B\sin\theta] + 3/4\sin\theta - 3/4\cos\theta$

Steady State equation

$y_p = 0$

$y_p' = 3/4 \cos t + 3/4 \sin\theta = 0$

$\therefore 3/4 \cos\theta + 3/4 \sin\theta = 0$

$\cancel{3/4}\cos\theta = -\cancel{3/4}\sin\theta$

$\frac{\cos\theta}{\cos\theta} = \frac{-\sin\theta}{\cos\theta}$

$1 = -\tan\theta$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\therefore \theta =$

Question 2

$EI\, d^2y/dx^2 = w/2\,(L-x)^2$

$EIm^2 = 0$

$m^2 = 0$

$m = \pm\sqrt{0}$

$m = \pm 0$

$y = e^{0.x}[A + Bx]$

$y = A + Bx$

$y_p = y = Fx^2 + Gx^3 + Hx^4$

$dy/dx = 2Fx + 3Gx^2 + 4Hx^3$

$d^2y/dx^2 = 2F + 6Gx + 12Hx^2$

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$$EI\left[2F + \cancel{3}6Gx + 12Hx^2\right] = \frac{\omega}{2}\left[L-x\right]^2$$

$$2FEI + \cancel{6}GEIx + 12HEIx^2 = \frac{\omega}{2}\left[L-x\right]^2$$

$$12GEIx$$

$$4FEI + \cancel{\ldots} + 24HEIx^2 = \omega\left[L^2 - 2Lx + x^2\right]$$

$$24HEI = \omega$$

$$H = \frac{\omega}{24EI} \quad \text{---} \quad ---(1)$$

$$12GEI = -2\omega L$$

$$G = \frac{-2\omega L}{12EI} = \frac{-\omega L}{6EI} \quad ---(2)$$

$$4FEI = \omega L^2$$

$$F = \frac{\omega L^2}{4EI}$$

$$y = \left[\frac{\omega L^2}{4EI}\right]x^2 - \left[\frac{\omega L}{6EI}\right]x^3 + \left[\frac{\omega}{24EI}\right]x^4$$

$$= \frac{\omega L^2 x^2}{4EI} - \frac{\omega x^2}{6EI} - \frac{\omega x^4}{24EI}$$

$$= \frac{6\omega L^2 x^2 - 4\omega L x^3 + 10x^4}{24EI}$$

$$G.F = y = A + Bx + \frac{\omega}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y=0$, $x=0$ $dy/dx = 0$

$$0 = A + B(0) + \frac{\omega}{24EI}\left(6L^2(0) + 4L(0)^3 + 0^4\right]$$

$$A = 0$$

$$dy/dx = B + \frac{\omega}{24EI}\left[12L^2x - 12Lx^2 + 4x^3\right]$$

$$0 = B + \frac{\omega}{24EI}\left[12L^2(0) - 12L(0)^2 + 4(0)^3\right]$$

$$B = 0$$

$$y_p = \frac{\omega}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$Y_P = \frac{wx^2}{24EI}\left[6L^2 - 4Lx + x^2\right]$$

$$Y_P = \frac{Wx^2}{24EI}\left[x^2 - 4Lx + 6L^2\right]$$

when $x = L$

$$Y_P = \frac{wL^2}{24EI}\left[L^2 - 4L^2 + 6L^2\right]$$

$$Y_P = \frac{WL^2}{24EI}\left[3L^2\right]$$

$$\therefore Y = \frac{wL^4}{8EI}$$