15/ENG02/003
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1) $y = e^{x^2+x}$ show that $y'' = y'(2x+1)+2y$

$y'' = y'(2x+1)+2y$

$y'' = \frac{\delta^2 y}{\delta x^2}$; $y' = \frac{\delta y}{\delta x}$

$\frac{\delta y}{\delta x} = (2x+1)e^{x^2+x}$

$\frac{\delta^2 y}{\delta x^2} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= \cancel{(4x^2+4x+1)e^{x^2}} + = (4x^2+4x+1)e^{x^2+x}$

$= (4x^2+4x^2+3)e^{x^2+2x}$

sub L.H.S and RHS

$(4x^2+4x+3)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x^2+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+3)e^{x^2+x}$

$= (4x^2+4x+4)e^{x^2+x}$

$(4x^2+4x+3)e^{x^2+x} = (4x^2+4x+3)e^{x^2+x}$

$\therefore y'' = y'(2x+1)+2y$

ii) $y'' = y'(2x+1)+2y$

$y'' - y'(2x+1) - 2y = 0$

let $y'' = w$

$V = 1$ $\quad V' = 0$

$U = y''$ $\quad U^n = y^{(n+2)}$

$w^n = U^n V + nU^{n-1}V'$

$= y^{(n+2)} + 0$

let $w = -y'(2x+1)$

$V = 2x+1$ $\quad V' = 2$ $\quad V'' = 0$

$U = -y'$ $\quad U^n = -y^{n+1}$

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$w^n = U^n V + nU^{n-1}V^1 + n(n-1)U^{n-2}V^{11}$

$= y^{n+1}(2x+1) + n(-y^{n+1})(2) + 0$

$= -y^{n+1}(2x+1) + 2n(-y^n)$

let $w = -2y$

$V = -2 \quad V^1 = 0$

$U = y \quad U^n = y^n$

$w^n = U^n V + nU^{n-1}V^1$

$= y^n \cdot -2 + 0$

$= -2y^n$

$\therefore y^n = y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^2$

$y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n-1)$

2.) $y = x^3 e^{4x}$ find y^5

$V = x^3;\ V^1 = 3x^2,\ V^{11} = 6x,\ V^{111} = 6,\ V^4 = 0$

$U^4 = 256e^{4x} \qquad U^5 = 1624e^{4x}$

$y^n = U^n V + nV^{n-1}V^1 + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3$

$+ \frac{n(n-1)(n-2)(n-3)}{4!}U^{n-4}V^4$

$y^5 = 1024e^{4x}(x^3) + 15x^2(256e^{4x}) + 60x(64e^{4x}) + 60(16e^{4x})$

$y^5 = x^3\, 24e^{4x} + x^2\, 3840e^{4x} + x\,3840e^{4x} + 960e^{4x}$

$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

a) $x^2 \frac{\delta y}{\delta x} + x\frac{\delta y}{\delta x} + y = 0$

$x^2 y^{11} + xy + y = 0$

let $w = x^2 y^{11}$

$V = x^2,\ V^1 = 2x,\ V^{11} = 2,\ V^{111} = 0$

$U = y^{11},\ U^n = y^{n+2}$

$w^n = U^n V + nU^{n-1}V^1 + \frac{n(n-1)}{2!}U^{n-2}V^{11} + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^{111}$

$= y^{n+2}(x^2) + n(y^{n+2})2x + \frac{n(n-1)}{2!}(y^{n+2-2})(2)$

$= x^2 y^{n+2} + 2xn(y^{n+1}) + n(n-1)y^n$

let $w = xy$

$v = x$, $v' = 1$, $v = 0$

$u = y$, $u^n = y^{n+1}$

$w^n = y^{n+1}(x) + x(y^{n+1-1})(1) + 1$

$= xy^{n+1} + ny^n$

let $w = y$

$v = 1$, $v' = 0$

$u = y$, $u^n = y^n$

$w^n = y^n$

$y^n = x^2 y^{(n+2)} + 2xn(y^{n+1}) + n(n-1)(y^n) + xy^{n+1} + ny^n + y$

$x^2 y(n+2) + (2n+1)x^{(n+1)} + (n^2 - n + n + 1)y^n = 0$

$x^2 y^{n+2} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

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