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MECHATRONICS
15/ENG05/040

1. $y = e^{x^2+x}$ —— (1)

$y' = (2x+1)e^{x^2+x}$ —— (1) $\quad y^n = a^n e^{ax}$

$y' \cdot u = 2x+1 \qquad v = e^{x^2+x}$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = (2x+1)e^{x^2+x}$

Using product rule

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

from eqn 1 & (11)

$y'' = y'(2x+1) + 2y$

$y^{(2)} = y^{(1)}(2x+1) + 2y$

$\text{LHS}_1 = y^{(2)} \qquad \text{LHS}_2 = y^{(1)}(2x+1)$

$u = y^{(2)} \quad v = 1 \qquad u = y^{(1)} \quad v = 2x+1$

$u^n = y^{(n+2)} \qquad u^n = y^{(1+n)} \quad v' = 2$

$u^{n-1} = y^{(n)}$

$\text{LHS}_3 = 2y$

$u = y \quad v = 2$

$u^n = y^n$

$\text{LHS}_1^{(n)} = \text{LHS}_2^{(n)} + \text{LHS}_3^{(n)}$

$y^n = u^n v + n u^{(n-1)} v'$

$y^{(2+n)} = y^{(1+n)} \cdot (2x+1) + n(y^{(n)}) \cdot 2 + y^{(n)} \cdot 2$

$y^{(2+n)} = (2x+1)y^{(1+n)} + 2(n+1)y^{(n)}$

2. $y = x^3 e^{4x}$ find y^5

$u = e^{4x} \quad v = x^3$

$u' = 4e^{4x} \quad v' = 3x^2$

$u'' = 16e^{4x} \quad v'' = 6x$

$u''' = 64e^{4x} \quad v''' = 6$

$u^{iv} = 256e^{4x} \quad v^{iv} = 0$

$u^{v} = 1024e^{4x} \quad v^{v} = 0$

$y^n = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v^2 + n\frac{(n-1)(n-2)}{3!}u^{(n-3)}v^3 + \cdots$

$y^5 = u^5 v + 5u^4 v' + 10u^{(3)}v^2 + 10u^{(2)}v^{(3)} + 5u'v^4 + uv^5$

$$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + \frac{5(4) 64 e^{4x}}{2!} + \frac{5(4)(3) 96 e^{4x}}{3!}$$

$$+ \frac{5(4)(3)(2) 4 e^{4x}(0)}{4!}$$

$$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} + 960 e^{4x} + 0$$

$$y^{(5)} = 64 e^{4x} [16x^3 + 60x^2 + 60x + 15]$$

2b $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$$x^2 y'' + x y' + y = 0$$

$w_1 = x^2 y''$

$u = y^2$ $\quad v = x^2$

$u^{(n)} = y^{(2+n)}$ $\quad v' = 2x$

$u^{(n-1)} = y^{(1+n)}$ $\quad v^2 = 2$

$u^{(n-2)} = y^{(n-1)}$ $\quad v^3 = 0$

$$w_1^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1) y^{(n)} \cdot 2}{2!} + 0$$

$$w_2^{(n)} = y^{(n+1)} x + n y^{(n)} \cdot 1 + 0$$

$$w_3^{(n)} = y^{(n)}$$

$$x^2 y(n+2) + 2xn y(n+1) + n(n-1) y^n + x y^{n+1} + n y^{(n)} + y^{(n)} = 0$$

$$x^2 y(n+2) + y^{n+1}(2xn + x) + y^n (n(n-1) + n + 1] = 0$$

$$x^2 y(n+2) + y^{(n+1)} [x(2n+1)] + y^n [n^2 - n + n + 1] = 0$$

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + y^n [n^2 + 1] = 0$$