

ENGA371 Assignment

15/ENGA02/003

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Solution

$$1) \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

Auxiliary equation - $m^2 - 2m - 2 = 0$

GS = CF + P.I

CF: Solve LHS = 0 $\therefore m^2 - 2m - 2 = 0$

$$\therefore (m^2 - 2m + m - 2) = 0$$

$$m(m-2) + 1(m-2) \therefore m = 2 \text{ or } -1$$

$$y = Ae^{-x} + Be^{2x}$$

P.I: $f(x) = 8$ Assume $y = C$

$$\therefore \frac{dy}{dx} = 0; \frac{d^2y}{dx^2} = 0$$

Substituting $\frac{dy}{dx} \rightarrow \frac{d^2y}{dx^2}$ in the given equation

$$0 - 0 - 2C = 8$$

$$\frac{-2C}{2} = \frac{8}{-2}$$

$$C = -4$$

$$PI = -4$$

GS = CF + P.I

$$\therefore y = Ae^{-x} + Be^{2x} - 4$$

$$2) \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

GS = CF + P.I

CF: Solve LHS: Auxiliary equation $m^2 - 4 = 0$

$$m^2 = 4; m = \pm 2$$

$$\therefore y = A \cosh 2x + B \sinh 2x$$

P.I: $f(x) = 10e^{3x}$ Assume $y = Ce^{3x}$

$$\frac{dy}{dx} = 3Ce^{3x}; \frac{d^2y}{dx^2} = 9Ce^{3x}$$

substitute $\frac{d^2y}{dx^2}$ and y in the given equation

$$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$5Ce^{3x} = 10e^{3x}$$

$$5C = 10$$

$$C = 2$$

$$y = A \cosh 2x + B \sinh 2x + 2e^{3x}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$GS = CF + PI$$

CF = solve LHS=0. Auxiliary equation $m^2 + 2m + 1 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - (4 \times 1 \times 1)}}{2 \times 1} = \frac{-2 \pm 0}{2}$$

$$m = -1 \text{ twice}$$

$$y = e^{-x}(A + Bx)$$

$$PI \Rightarrow f(x) = e^{-2x} \therefore \text{Assume } y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}, \quad \frac{d^2y}{dx^2} = 4Ce^{-2x}$$

Substitute the value of $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$\therefore y = e^{-2x}$$

$$GS = CF + PI$$

$$y = e^{-x}(A + Bx) + e^{-2x}$$

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$GS = CF + PI$$

CF \Rightarrow solve LHS=0 \therefore Auxillary equation $m^2 + 25 = 0$

$$m^2 = -25 \quad \therefore m = \pm j5$$

$$\therefore y = A \cos 5x + B \sin 5x$$

PI $\Rightarrow f(x) = 5x^2 + x \quad \therefore$ Assume $y = Cx^2 + Dx + E$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

Substituted $\frac{d^2y}{dx^2}$ and y in the equation

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$$x^2: \quad 25C = 5$$

$$C = \frac{1}{5}$$

$$x: \quad 25D = 1$$

$$D = \frac{1}{25}$$

$$x^0: \quad 2C + 25E = 0$$

$$2\left(\frac{1}{5}\right) + 25E = 0$$

$$25E = -\frac{2}{5}$$

$$E = -\frac{2}{125}$$

$$\therefore y = \left(\frac{1}{5}x^2\right) + \left(\frac{1}{25}x\right) + \left(-\frac{2}{125}\right)$$

$$y = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$$

$$y = \frac{25x^2 + 5x - 2}{125}$$

$$GS = CF + PI$$

$$GS = A \cos 5x + B \sin 5x + \frac{25x^2 + 5x - 2}{125}$$

$$5) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4 \sin x$$

$$GS = CF + PI$$

CF \Rightarrow solve LHS=0 \therefore Auxillary equation $m^2 - 2m + 1 = 0$

$$m^2 - m - m + 1 = 0 \Rightarrow m(m-1) - 1(m-1) = 0$$

$$m = -1 \text{ twice}$$

$$y = e^{-x}(A + Bx)$$

PI $\Rightarrow f(x) = 4 \sin x \quad \therefore$ Assume $y = C \cos x + D \sin x$

$$dy/dx = -C \sin x + D \cos x$$

$$d^2y/dx^2 = -C \cos x - D \sin x$$

Substitute into the general equation

$$-C \cos x - D \sin x - 2(-C \sin x + D \cos x) + C \cos x + D \sin x = 4 \sin x$$

$$-C \cos x - D \sin x + 2C \sin x - 2D \cos x + C \cos x + D \sin x = 4 \sin x$$

$$(-D + 2C + D) \sin x + (-C - 2D + C) \cos x = 4 \sin x$$

$$\sin x: -D + 2C + D = 4$$

$$\frac{2C}{2} = \frac{4}{2} \quad \therefore C = 2$$

$$\cos x: -C - 2D + C = 0$$

$$-2 - 2D + 2 = 0$$

$$-2D = 0$$

$$D = 0$$

$$y = 2 \cos x + 0 \sin x$$

$$y = 2 \cos x$$

$$GS = CF + PI \quad \therefore GS = e^{-2x}(A + Bx) + 2 \cos x$$

$$6) \quad d^2y/dx^2 + 4 dy/dx + 5y = 2e^{-2x}$$

$$\text{CF} \Rightarrow \text{Solve } CHS = 0 \quad m^2 + 4m + 5 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2} = -2 \pm j$$

$$PI \Rightarrow f(x) = 2e^{-2x} \quad \therefore y = e^{-2x}(A \cos x + B \sin x)$$

$$\frac{dy}{dx} = -2ce^{-2x}; \quad \frac{d^2y}{dx^2} = 4ce^{-2x}$$

Substitute in the equation

$$4ce^{-2x} + 4(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$$

$$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$$

$$e^{-2x}: 4c - 8c + 5c = 2$$

$$c = 2$$

$$y = 2e^{-2x}$$

$$(GS) y = e^{-2x}(A \cos x + B \sin x) + 2e^{-2x}$$

$$x=0; y=1$$

$$1 = A + 2$$

$$A = -1$$

$$y = e^{-2x}(-\cos x + B\sin x) + 2e^{-2x}$$

$$\frac{dy}{dx} = e^{-2x}(\sin x + B\cos x) - 2e^{-2x}(-\cos x + B\sin x) - 4e^{-2x}$$

$$\text{If } x=0 \text{ and } \frac{dy}{dx} = -2$$

$$-2 = B + 2 - 4$$

$$B = 0$$

$$\text{Particular solution is } y = e^{-2x}(-\cos x) + 2e^{-2x}$$

$$y = -e^{-2x}\cos x + 2e^{-2x}$$

$$y = e^{-2x}(2 - \cos x)$$

$$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

$$\text{CF} \Rightarrow \text{LHS} = 0 \therefore \text{Auxillary equation} = 3m^2 - 2m - 1 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{2 \pm \sqrt{(-2)^2 - (4 \times 3 \times -1)}}{2 \times 3}$$

$$= \frac{2 \pm \sqrt{4 + 12}}{6} = \frac{2 \pm 4}{6}$$

$$m = 3 \text{ or } -1$$

$$\therefore y = Ae^{-x} + Be^{3x}$$

$$\text{PI} \div f(x) = 2x - 3$$

$$\text{By } y = Cx + D$$

$$\frac{dy}{dx} = C; \frac{d^2y}{dx^2} = 0$$

Substitute into the given equation

$$3(0) - 2(C) - (Cx + D) = 2x - 3$$

$$-2C - (Cx + D) = 2x - 3$$

$$x: -C = 2$$

$$C = -2$$

$$x^0: -2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$7 = D$$

$$\therefore y = -2x + 7$$

$$GS = CF + PI$$

$$GS = Ae^{-x} + Be^{3x} - 2x + 7$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$m^2 - 6m + 8 = 0 \quad \div \text{ (Auxillary equation)}$$

$$CF \Rightarrow LHS = 0$$

$$m^2 - 4m - 2m + 8 = 0$$

$$m(m-4) - 2(m-4) = 0$$

$$m = 4 \text{ or } 2$$

$$y = Ae^{2x} + Be^{4x}$$

$$PI \Rightarrow f(x) = 8e^{4x} \quad \text{But } y = Ce^{4x}$$

$$\frac{dy}{dx} = 4Ce^{4x} ; \quad \frac{d^2y}{dx^2} = 16Ce^{4x}$$

Substitute $\frac{d^2y}{dx^2}$ into the general equation

$$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$$

$$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$$

$$e^{4x}: 16C - 24C + 8C = 8$$