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COLLEGE OF ENGINEERING

DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING

PROCESS DYNAMICS & CONTROL

CHE 531 ASSIGNMENT

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$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 2e^{3t}$$

$$y'' - 3y' + 2y = 2e^{3t}$$

Functions and their Laplace Transforms

$$e^{at} = \frac{1}{s-a}$$

$$y = y(s)$$

$$y'(t) = sy(s) - y(0)$$

$$y''(t) = s^2y(s) - sy(0) - y'(0)$$

Therefore:

$$s^2y(s) - sy(0) - y'(0) - 3\big(sy(s) - y(0)\big) + 2y(s) = 2\left(\frac{1}{s-a}\right)$$

$$s^2y(s) - sy(0) - y'(0) - 3sy(s) + 3y(0) + 2y(s) = \frac{2}{s-a}$$

At:

$$y(0) = 5 \qquad y'(0) = 7$$

$$s^2y(s) - s(5) - 7 - 3sy(s) + 3(5) + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 5s - 7 - 3sy(s) + 15 + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 3sy(s) + 2y(s) + 5S + 8 = \frac{2}{s-3}$$

$$y(s)(s^2 - 3s + 2) - 5S + 8 = \frac{2}{s-3}$$

$$y(s)(s^2 - 3s + 2) = \frac{2}{s-3} + 5S - 8$$

$$y(s)(s^2 - 3s + 2) = \frac{2 + 5s(s-3) - 8(s-3)}{s-3}$$

$$y(s) = \frac{2 + 5s(s-3) - 8(s-3)}{s-3}\left(\frac{1}{(s^2 - 3s + 2)}\right)$$

$$y(s) = \frac{2 + 5s(s-3) - 8(s-3)}{(s-3)(s^2 - 3s + 2)}$$

$$y(s) = \frac{2 + 5s(s-3) - 8(s-3)}{(s-3)(s^2 - 3s + 2)} = \frac{2 + 5s(s-3) - 8(s-3)}{(s-3)(s-2)(s-1)}$$

By Partial Fraction Expansion

$$\frac{2 + 5s(s-3) - 8(s-3)}{(s-3)(s-2)(s-1)} = \frac{A}{(s-3)} + \frac{B}{(s-2)} + \frac{C}{(s-1)}$$

$$\frac{2 + 5s(s-3) - 8(s-3)}{(s-3)(s-2)(s-1)} = \frac{A(s-2)(s-1) + B(s-3)(s-1) + C(s-3)(s-2)}{(s-3)(s-2)(s-1)}$$

$$2 + 5s(s-3) - 8(s-3) = A(s-2)(s-1) + B(s-3)(s-1) + C(s-3)(s-2)$$

Let $S = 1$

$$2 + 5(1)(1-3) - 8(1-3) = A(1-2)(1-1) + B(1-3)(1-1) + C(1-3)(1-2)$$

$$2 - 10 + 16 = A(-1)(0) + B(-2)(0) + C(-2)(-1)$$

$$8 = 2C$$

$$C = \frac{8}{2} = 4$$

Let $S = 2$

$$2 + 5(2)(2-3) - 8(2-3) = A(2-2)(2-1) + B(2-3)(2-1) + C(2-3)(2-2)$$

$$2 - 10 + 8 = A(0)(1) + B(-1)(1) + C(-1)(-0)$$

$$0 = -B$$

$$B = \frac{0}{-1} = 0$$

Let $S = 3$

$$2 + 5(3)(3-3) - 8(3-3) = A(3-2)(3-1) + B(3-3)(3-1) + C(3-3)(3-2)$$

$$2 - 0 - 0 = A(1)(2) + B(0)(2) + C(0)(1)$$

$$2 = 2A$$

$$A = \frac{2}{2} = 1$$

$$\frac{2 + 5s(s-3) - 8(s-3)}{(s-3)(s-2)(s-1)} = \frac{1}{(s-3)} + \frac{0}{(s-2)} + \frac{4}{(s-1)}$$

$$y(s) = \frac{2 + 5s(s-3) - 8(s-3)}{(s-3)(s-2)(s-1)} = \frac{1}{(s-3)} + \frac{4}{(s-1)}$$

Taking the inverse Laplace of y(s) will give:

$$y(t) = \boldsymbol{\mathcal{L}^{-1}}\{\boldsymbol{y(s)}\} = \boldsymbol{\mathcal{L}^{-1}}\left\{\frac{1}{(s-3)} + \frac{4}{(s-1)}\right\}$$

$$y(t) = \boldsymbol{\mathcal{L}^{-1}}\left\{\frac{1}{(s-3)}\right\} + \boldsymbol{\mathcal{L}^{-1}}\left\{\frac{4}{(s-1)}\right\}$$

$$y(t) = \boldsymbol{\mathcal{L}^{-1}}\left\{\frac{1}{S-3}\right\} + (4)\boldsymbol{\mathcal{L}^{-1}}\left\{\frac{1}{S-1}\right\}$$

$$y(t) = e^{3t} + 4e^{t}$$