

$$1. \frac{dv}{dt} = F_i - F$$

Since  $F = B\sqrt{h}$

$$\frac{d(Ah)}{dt} = F_i - B\sqrt{h}$$

$$A \frac{dh}{dt} = F_i - B\sqrt{h} \quad \text{--- (1)}$$

Linearizing  $B\sqrt{h}$ , we have

$$F(x) = F(x_s) + \left. \frac{df}{dx} \right|_{x=x_s} (x - x_s)$$

$$B\sqrt{h} = B\sqrt{h_s} + \frac{1}{2\sqrt{h_s}} B(h - h_s)$$

$$A \frac{dh}{dt} = F_i - B\sqrt{h_s} + \frac{1}{2\sqrt{h_s}} B(h - h_s)$$

$$A \frac{dh}{dt} = F_i - B\sqrt{h_s} + \frac{B}{2\sqrt{h_s}} (h - h_s) \quad \text{--- (2)}$$

At steady state, eqn (1) with becomes

$$A \frac{dh_s}{dt} = F_{i_s} - B\sqrt{h_s} \quad \text{--- (3)}$$

Subtracting (2) from eqn (3), we have

$$A \frac{dh}{dt} - A \frac{dh_s}{dt} + \left( B\sqrt{h_s} + \frac{B}{2\sqrt{h_s}} (h - h_s) - B\sqrt{h_s} \right) = (F_i - F_{i_s})$$

$$A \frac{d(h - h_s)}{dt} + \frac{B}{2\sqrt{h_s}} (h - h_s) = F_i - F_{i_s}$$

$$A \frac{d\bar{h}}{dt} + \frac{B}{2\sqrt{h_s}} \bar{h} = \bar{F}_i$$

$$A \left[ s\bar{h}(s) - \bar{h}(0) \right] + \frac{B}{2\sqrt{h_s}} \bar{h}_s = \bar{F}_i$$

$$A s\bar{h}(s) + \frac{B}{2\sqrt{h_s}} \bar{h}_s = \bar{F}_i$$

$$\bar{h}_s \left( \frac{A_s + B}{2\sqrt{h}_s} \right) = \bar{F}_i$$

$$G(s) = \frac{h(s)}{F_i} = \frac{1}{A_s + \frac{B}{2\sqrt{h}_s}}$$

$$= \frac{1}{\frac{A_s 2\sqrt{h}_s + B}{2\sqrt{h}_s}}$$

$$= \frac{2\sqrt{h}_s}{A_s 2\sqrt{h}_s + B} \quad \text{--- (4)}$$

Equ (4) is the transfer function

$$2. G_p(s) = \frac{2}{s+1}$$

$$G_p(s) = G_m(s) = 1$$

$$k_c = 1.5$$

Using P only controller

$$Y(s) = \frac{G_p(s) G_f(s) G_c(s)}{1 + G_p(s) G_f(s) G_m(s)}$$

$$Y(s) = \frac{G_p(s) G_c(s)}{1 + G_p(s) G_c(s)} \times Y_r(s)$$

$$Y(s) = \frac{2/s+1 \cdot 1.5}{1 + 2/s+1 \cdot 1.5} \times \frac{3}{s}$$

$$Y(s) = \frac{3/s+1}{1 + 3/s+1} \times \frac{3}{s}$$

$$= \frac{3}{s+1} \times \frac{3}{s} \\ = \frac{3}{s+1} \times \frac{3}{s}$$

$$= \frac{3}{s+1} - \frac{s+1+3}{s+1} \times \frac{3}{s}$$

$$= \frac{3}{s+4} - \frac{3}{s}$$

$$Y(s) = \frac{9}{s(s+4)}$$

$$Y(s) = \frac{9}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$$Y(s) = \frac{9}{s(s+4)} = \frac{A(s+4)}{s(s+4)} + \frac{Bs}{s(s+4)}$$

$$Y(s) = 9 = A(s+4) + Bs$$

When  $s=0$

$$9 = A(5(0)+4) + B(0)$$

$$A = \frac{9}{4}$$

When  $s = -4/5$

$$9 = A(5(-4/5)+4) + B(-4/5)$$

$$9 = A(-4+4) + B(-4/5)$$

$$9 = -4/5 B$$

$$B = -45/4$$

$$\therefore \frac{9}{s(s+4)} = \frac{9}{4} \frac{1}{s} - \frac{45}{4} \frac{1}{(s+4)}$$

$$Y(s) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{9}{4s} - \frac{45}{4(s+4)}\right]$$

$$\mathcal{L}^{-1}\left[\frac{9}{4s}\right] - \mathcal{L}^{-1}\left[\frac{45}{4(s+1)}\right]; \mathcal{L}^{-1}\left[\frac{9}{4s}\right] - \mathcal{L}^{-1}\left[\frac{45}{4 \times 5(s+4/5)}\right]$$

$$\frac{9}{4} \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \frac{45}{20} \mathcal{L}^{-1}\left[\frac{1}{s+4/5}\right]$$

$$Y(s) = \frac{9}{4} \cdot 1 - \frac{9}{4} e^{-4/5t}$$

$$y(t) = \frac{9}{4} - \frac{9}{4} e^{-4/5t}$$

$$y(t) = \frac{9}{4} \left( 1 - e^{-4/5t} \right)$$