Name: Osigele-Kehinde

matric no: 16/ENG02/049

Department: Computer Engineering

## Assignment

1a $\lim_{x \to \pi/2} \left[ \dfrac{x^2 - \pi/4 + \sin(\cos x)}{x - \pi/2} \right]$

$= \lim_{x \to \pi/2} \left[ \dfrac{(2x - 0)(\sin)(-\sin x) + \cos x (\cos x)}{1 - 0} \right]$

$= \lim_{x \to \pi/2} \left[ \dfrac{2x(-\sin 2x + \cos 2x)}{1} \right]$

$= 2 \cdot \pi/2 \left( -\sin^2(\pi/2) + \cos^2(\pi/2) \right)$

$= \pi(-1 + 0)$

$= -\pi$

$\lim_{x \to \pi/2}$

1b $\lim_{x \to \pi/2} \ln \left[ \exp \dfrac{(3x^2 + 2x - 1)}{x + 1} \right]$

$= \lim_{x \to \pi/2} \left[ \dfrac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}} \right]$

$= \lim_{x \to \pi/2} (3x - 1)$

$= 3(\pi/2) - 1$

$= 3\pi/2 - 1$

1c $\lim_{x \to 2+\sqrt{3}} \cos \left[ \dfrac{\sin^{-1}(x-2)}{(x - \sqrt{3})} \right]$

$= \lim_{x \to 2+\sqrt{3}} \dfrac{\cos \sin^{-1}(x-2)}{\cos \sin^{-1}(x - \sqrt{3})}$

$= \cos \left[ \sin^{-1} \dfrac{(\cancel{2} + \sqrt{3} - \cancel{2})}{(2 + \sqrt{3} - \sqrt{3})} \right]$

$= \cos \left[ \sin^{-1} \dfrac{\sqrt{3}}{2} \right]$

$= \cos 60 \quad = 1/2$

1d $\lim_{x \to 4} \left[ \dfrac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$\lim_{x \to 4} \left( \dfrac{x^2 - 4x - 4x + 16}{x^2 - 4 - x + 4} \right)$

$\lim_{x \to 4} \left[ \dfrac{(x^2 - 4x) - (4x + 16)}{(x^2 - 4x) - (x - 4)} \right]$

$= \lim_{x \to 4} \left[ \dfrac{x(x-4) - 4(x-4)}{x(x-4) - 1(x-4)} \right]$

$= \lim_{x \to 4} \left[ \frac{x-4}{x-1} \right]$

$\frac{4-4}{4-1} = \frac{0}{1} = 0$

2a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \dots$

$U_n = \frac{2}{(n+1)\times(n+2)}$ $U_{n+1} = \frac{2}{(n+2)\times(n+3)}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{\cancel{2}}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{\cancel{2}}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{n^2+3n+2}{n^2+5n+6}$

$= \lim_{n \to \infty} = \frac{n^2/n^2 + 3n/n^2 + 2/n^2}{n^2/n^2 + 5/n^2 + 6/n^2}$

$\frac{1 + 3n/n^2 + 2/n^2}{1 + 5/n^2 + 6/n^2}$

$= \frac{1+0+0}{1+0+0}$

From test 1 $\lim_{n \to \infty} U_n = \lim_{n \to \infty} \frac{2}{(n^2+3n+2)}$

$= \frac{2/n^2}{n^2/n^2 + 3n/n^2 + 2/n^2}$

$= \frac{0}{1+0+0}$

$= 0$ The series is convergent

2c $U_n = \frac{1+2n^2}{1+n^2}$

$\lim U_n = 0$

$\lim_{n \to \infty} \frac{1+2n^2}{1+n^2} = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}$

$n \to \infty \; 1/n^2 = 0$

$\frac{0+2}{0+1} = 2$ The series is divergent

Scanned by CamScanner

2b $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

$U_n = \frac{2}{n^2}$, $U_{n+1} = \frac{2}{(n+1)^2}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \times \frac{n^2}{2}$

$= \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1}$

divide by the highest power of n

$= \frac{n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2}$

$= \frac{1}{1+0+0} = \frac{1}{1}$

Using test 1 on $U_n$

$U_n = \frac{2}{n^2}$

$= \frac{2/n^2}{n^2/n^2}$

$= 0/1$

3 $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

$U_n = \frac{x^n}{(2n+1)^3}$, $U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$

$\lim_{n\to\infty} = \frac{U_{n+1}}{U_n} = \frac{\cancel{x^n} \times x^1}{(2n+3)^3} \times \frac{(2n+1)^3}{\cancel{x^n}}$

$\frac{x^1(2n+1)^3}{(2n+3)^3}$

$= \frac{(8n^3+12n^2+6n+1)x}{8n^3+18n^2+54n+27}$

divide by $n^3$

$\frac{8xn^3/n^3 + 12xn^2/n^3 + 6x/n^3 + x/n^3}{8n^3/n^3 + 18n^2/n^3 + 54/n^3 + 27/n^3}$

$\frac{8x+0+0+0}{8+0+0+0} = \frac{8x}{8}$ $\frac{8x}{8} < 1$, $8x < 8$, $x < 1$

Scanned by CamScanner

4 $\lim_{x \to 0} \left[ \frac{\sin x - \cos x}{x^3} \right]$

$\lim_{x \to 0} \left[ \frac{\sin(0) - \cos(0)}{(0)^3} \right] = \lim_{x \to 0} \left[ \frac{0 - 1}{0} \right] = \frac{-1}{0} = 0_{//}$

Apply L'Hospital

$\lim_{x \to 0} \left[ \frac{\cos(0) + \sin(0)}{3(0)^2} \right] = \frac{1}{0} = 0_{//}$

$\lim_{x \to 0} \left[ \frac{-\sin(0) + \cos(0)}{6(0)} \right] = \frac{1}{0} = 0_{//}$

$\lim_{x \to 0} \left[ \frac{-\cos(0) - \sin(0)}{6} \right] = -1/6_{//}$

Scanned by CamScanner