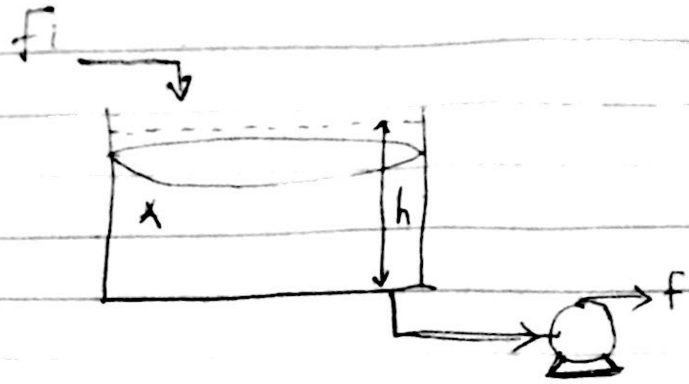
1)



$x/B \quad V = Ah$

$f = B\sqrt{h}$

$$\frac{dV}{dt} = f_i - f.$$

$$\frac{d(Ah)}{dt} = f_i - B\sqrt{h}$$

$$A\frac{dh}{dt} = f_i - \beta\sqrt{h} \quad \cdots (1)$$

for steady state

$$A\frac{dh_s}{dt} = f_{is} - \beta\sqrt{h_s} \longrightarrow (2)$$

$$A\frac{dh_s}{dt} + \beta\sqrt{h_s} = f_{is} \longrightarrow (3)$$

$B\sqrt{h}$ is not a linear so $B\sqrt{h}$ has to be linearized ~~used~~ using the linearization formula of linearization for a single variable

$$f(x) = f(x_s) + \left.\frac{df}{dx}\right|_{x=x_s}(x - x_s)$$

$$f(B\sqrt{h}) = B\sqrt{h_s} + \frac{1}{2\sqrt{h_s}}.B(h - h_s) \text{ --- } (4)$$

f. Putting (4) into (1)

$$A\frac{dh}{dt} = f_i - (B\sqrt{h_s} + \frac{B}{2\sqrt{h_s}}(h - h_s)$$

$$A\frac{dh}{dt} + \frac{B\sqrt{h_s}}{2} + \frac{B}{2\sqrt{h_s}}(h - h_s) = f_i \text{ --- } (5)$$

Substract (3) from (1)

$$A\frac{dh}{dt} + \frac{B}{2}\sqrt{h_s} + \frac{B}{2\sqrt{h_s}} = f_i$$

$$= A\frac{d\bar{h}}{dt} + \frac{B\bar{h}}{2\sqrt{hs}} = \bar{f}_i$$

Taking Laplace of both Side

$$A\mathcal{L}^{-1}\left[\frac{d\bar{h}}{dt}\right] + \frac{B}{2\sqrt{hs}}\mathcal{L}^{-1}[\bar{h}] = \mathcal{L}^{-1}[\bar{f}_i]$$

$$A(S\bar{h} - \cancel{h(0)}^{0}) + \frac{B\bar{h}(s)}{2\sqrt{hs}} = \bar{f}_i(s)$$

$$A s\bar{h}(s) + \frac{B\bar{h}(s)}{2\sqrt{hs}} = \bar{f}_i(s)$$

$$G_P(s) = \frac{\text{Output}}{\text{Input}}$$

$$G_P(s) = \bar{h}s\left[As + \frac{B}{2\sqrt{hs}}\right] = \bar{f}_i(s)$$

$$G_P(s) = \frac{\bar{h}(s)}{\bar{f}_i(s)} = \frac{1}{\left(As + \frac{B}{2\sqrt{hs}}\right)}$$

$$\cancel{G_P}\; G_P(s) = \frac{1}{As + \frac{B}{2\sqrt{hs}}}$$

$$G_P(s) = \frac{2\sqrt{hs}}{2A\sqrt{hs}\,s + B} = \frac{2\sqrt{4}}{2(25)(\sqrt{4}s + 2)} = \frac{4}{10s+2} = \frac{2}{5s+1}$$

$$\therefore G_P(s) = \frac{2}{5s+1}$$

Given $G_p(s) = \frac{2}{5s+1}$, $G_f(s) = G_m(s) = 1$, $K_c = 1.5$, 3 step unit change

$$y(s) = \frac{G_p(s)\,G_f(s)\,G_c(s)}{1 + G_p(s) + G_f(s) + G_c(s) + G_m(s)} \cdot y_{sp}s$$

$$y(s) = \frac{\frac{2}{5s+1}(1.5)}{1 + \frac{2}{5s+1}(1.5)} \cdot \frac{3}{s}$$

$$y(s) = \frac{\frac{3}{5s+1}}{1 + 3/5s+1} \cdot 3/s$$

$$y(s) = \frac{\frac{3}{5s+1}}{\frac{5s+1+3}{5s+1}} \cdot \frac{3}{5} = \frac{\cancel{3}}{\cancel{5s+1}} \times \frac{\cancel{5s+1}}{5s+4}$$

$$= \frac{3}{5s+4} \cdot \frac{s}{3} = \frac{9}{s(5s+4)}$$

$$y(s) = \frac{9}{s(5s+4)} = \frac{A}{s} + \frac{B}{5s+4}$$

let $s = 0$

$$\therefore = A(5(0)+4 + B(0))$$

$$9 = 4A$$

$$A = 9/4 = 2\,1/4$$

let $s = -4/5$

$$9 = A\left(5\left(-\frac{4}{5}\right)+4\right) - 4\frac{B}{5}$$

$$9 = -\frac{4B}{5}$$

$$B = \frac{45}{4}$$

$$B = -\frac{45}{4}$$

$$\frac{9}{s(5s+4)} = \frac{9}{4s} - \frac{45}{4(5s+4)}$$

$$\mathcal{L}^{-1}\left(\frac{9}{4s}\right) - \mathcal{L}^{-1}\left[\frac{45}{4(5s+4)}\right]$$

$$y(t) = \frac{9}{4}\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \frac{45}{20}\mathcal{L}^{-1}\left[\frac{1}{s+4/5}\right]$$

$$y(t) = \frac{9}{4} - \frac{45}{20}e^{-4/5t}$$

$$y(t) = \frac{9}{4}\left(1 - e^{-4/5t}\right)$$